



Soongsil University

Random Walk with Restart on Hypergraphs: Fast Computation and an Application to Anomaly Detection



Jaewan Chun



Geon Lee



Kijung Shin



Jinhong Jung

Group Interactions are Everywhere

- Example 1: Co-authorship of researchers

How Do Hyperedges Overlap in Real-World Hypergraphs? - Patterns, Measures, and Generators

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Hypercore Decomposition for Non-Fragile Hyperedges: Concepts, Algorithms, Observations, and Applications

Fanchen Bu*, Geon Lee†, and Kijung Shin‡

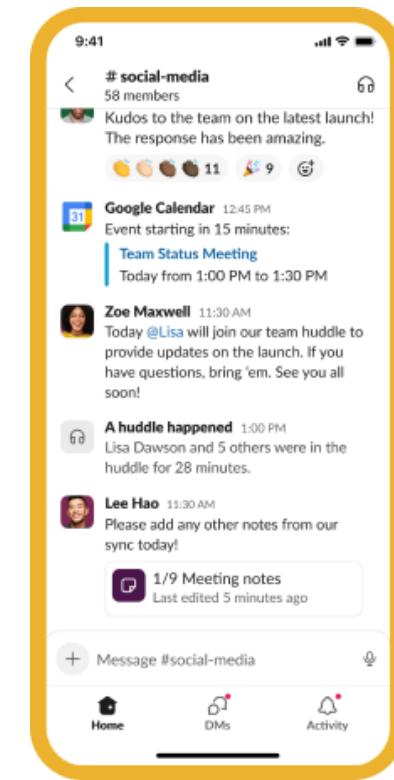
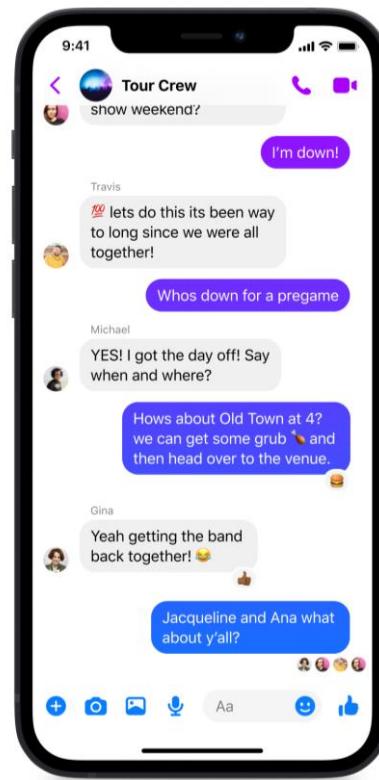
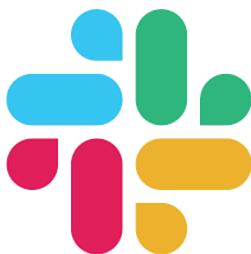
MiDaS: Representative Sampling from Real-world Hypergraphs

Minyoung Choe KAIST minyoung.choe@kaist.ac.kr	Jaemin Yoo Seoul National University jaeminyoo@snu.ac.kr	Geon Lee KAIST geonlee0325@kaist.ac.kr
Woonsung Baek KAIST wbaek@kaist.ac.kr	U Kang Seoul National University ukang@snu.ac.kr	Kijung Shin* KAIST kijungs@kaist.ac.kr



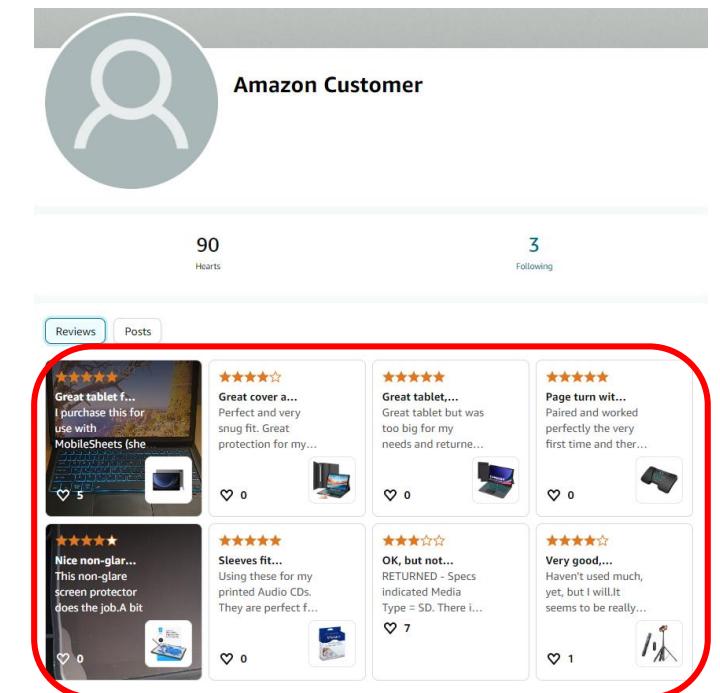
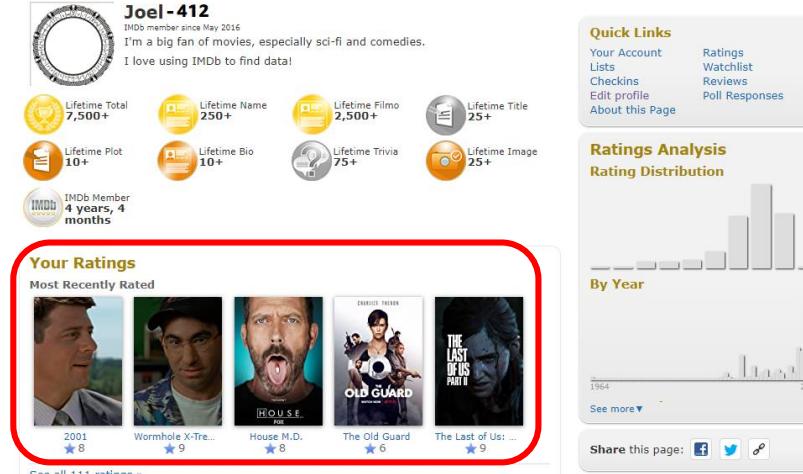
Group Interactions are Everywhere (cont.)

- Example 2: Online group chats



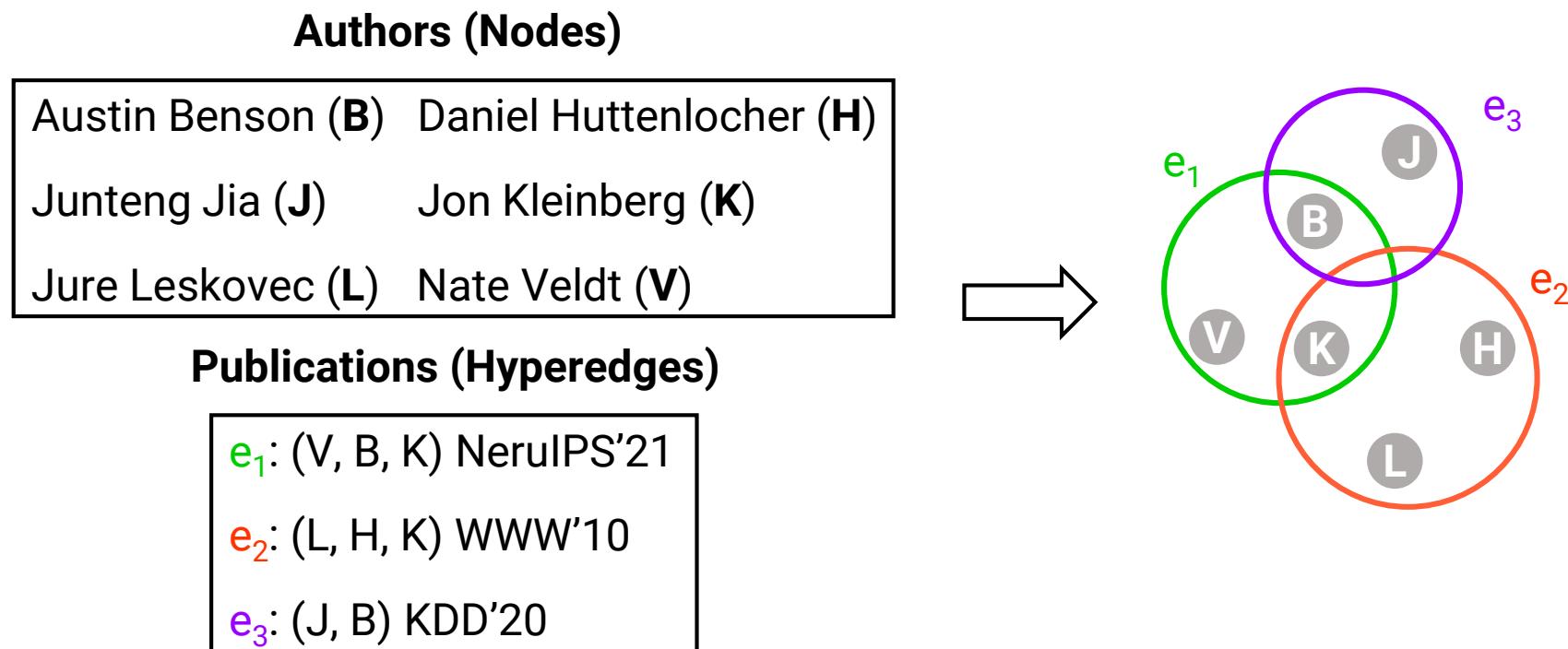
Group Interactions are Everywhere (cont.)

- Example 3: Reviewed item set



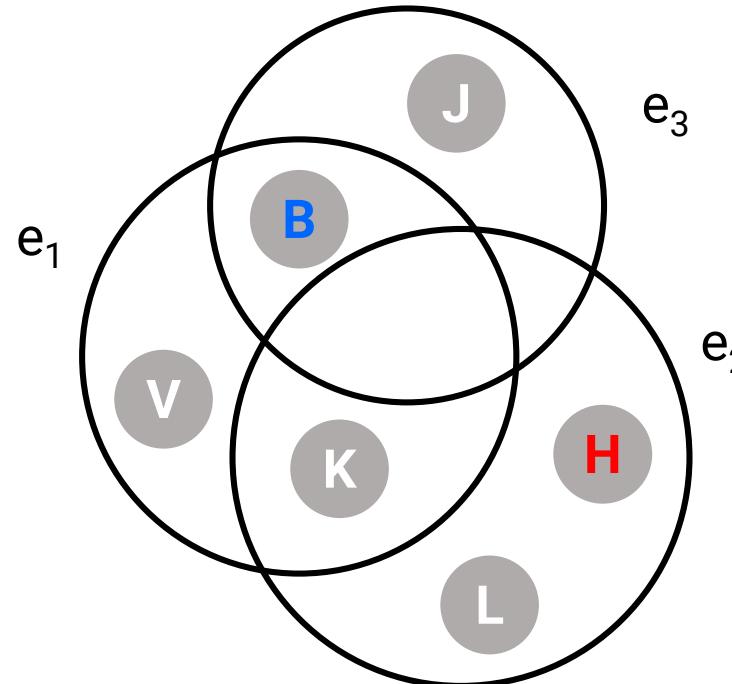
Hypergraphs: Model for Group Interactions

- **Hypergraphs** model group interactions among individuals or objects
- Each **hyperedge** is a subset of any number of nodes



Measuring Proximity between Nodes

- Measuring **proximity** between nodes in a hypergraph:
 - How **close are two nodes** in a hypergraph to each other?



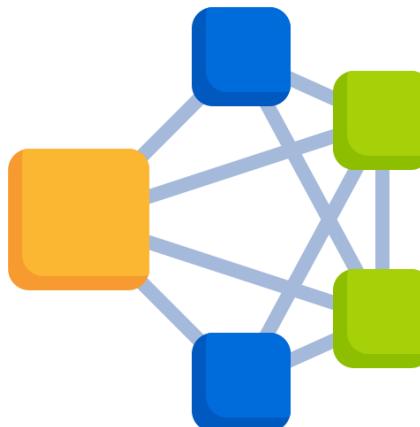
*How close is node H to node B ?
That is, what is the proximity of H w.r.t. B ?*

Measuring Proximity between Nodes (cont.)

- Measuring **proximity** between nodes in a hypergraph
 - How **close** are two nodes in a hypergraph to each other?
- Practical applications of measuring node proximity:



Anomaly Detection



Clustering



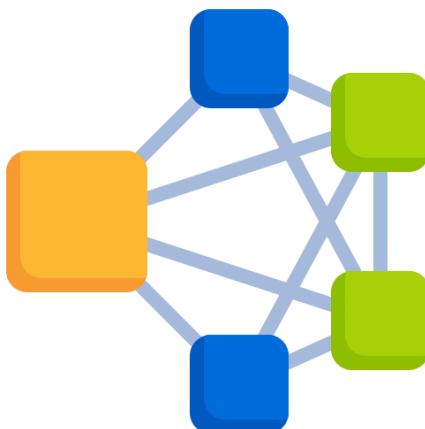
Personalized Ranking

Measuring Proximity between Nodes: RWR

- Widely used method: **Random Walk with Restart (RWR)**
 - Proposed for graphs → extended to hypergraphs
 - Consider global network structure and multi-faceted relationship
- Successful applications of RWR:



Anomaly Detection
(ICDM'05)



Clustering
(CIKM'20)



Personalized Ranking
(ICDM'16)

RWR on Hypergraphs?

- RWR on hypergraphs has been **underexplored**
- Potential reason: lack of **fast** computation methods!
 - Various computation methods exist only for **graphs**
 - Iterative methods
 - Power iteration (Page'99)
 - Preprocessing methods
 - BEAR (SIGMOD'15)
 - BePI (SIGMOD'17)
 - Approximation-based methods
 - Spectral sparsification (ICESS'19)
 - Eulerian Laplacian approximation (FOCS'16)

RWR on Hypergraphs? (cont.)

- RWR on hypergraphs has been **underexplored**
- Potential reason: lack of **fast** computation methods!

Our goal: fast RWR computation method for hypergraphs

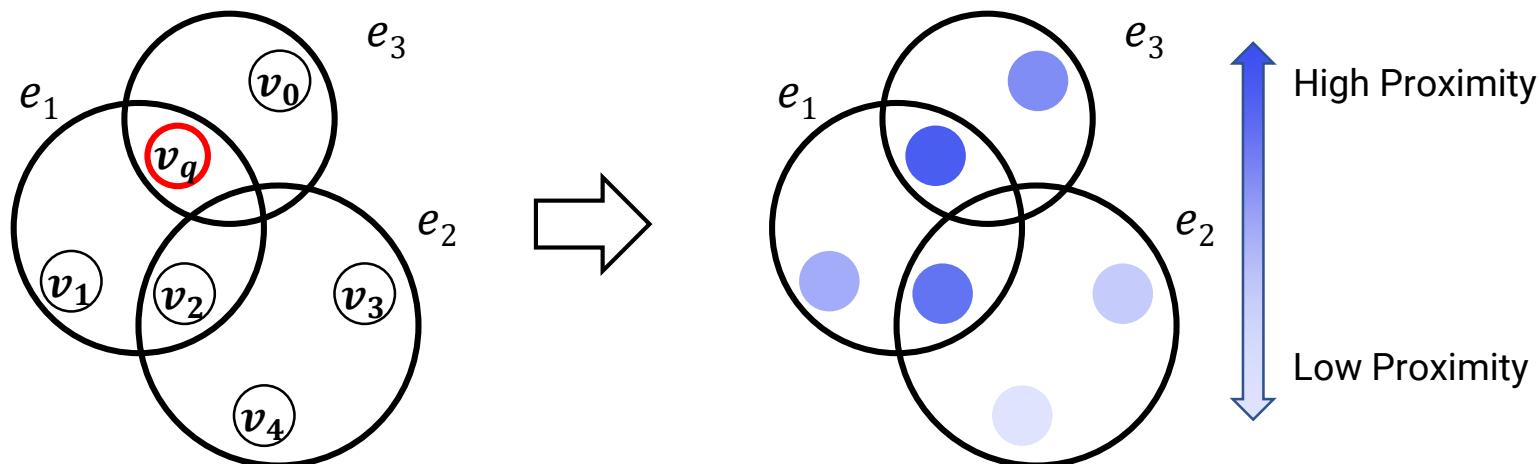
Roadmap

- Overview
- Random Walk with Restart (RWR) on Hypergraphs <<
- Proposed Algorithm: ARCHER
- Empirical Evaluation of ARCHER
- Application: Anomaly Detection
- Conclusion



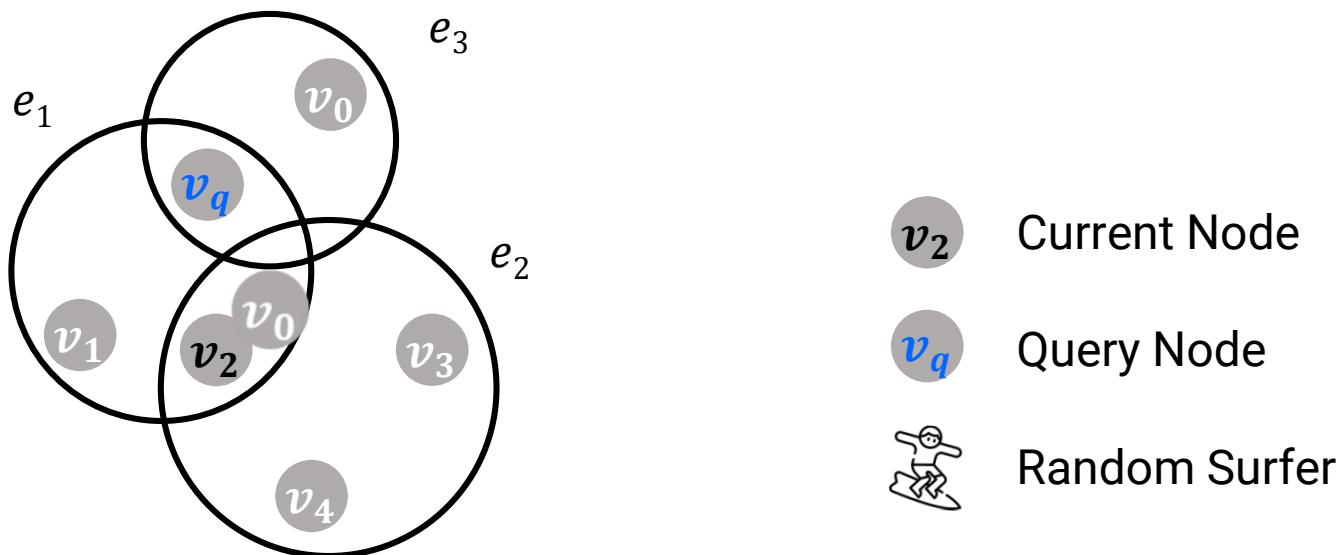
RWR on Hypergraphs: Inputs and Outputs

- **Given**
 - Hypergraph $G_H = (V, E)$
 - Node set V and hyperedge set E
 - Query node v_q
 - Restart probability c
- **Output:** proximity between each node and v_q



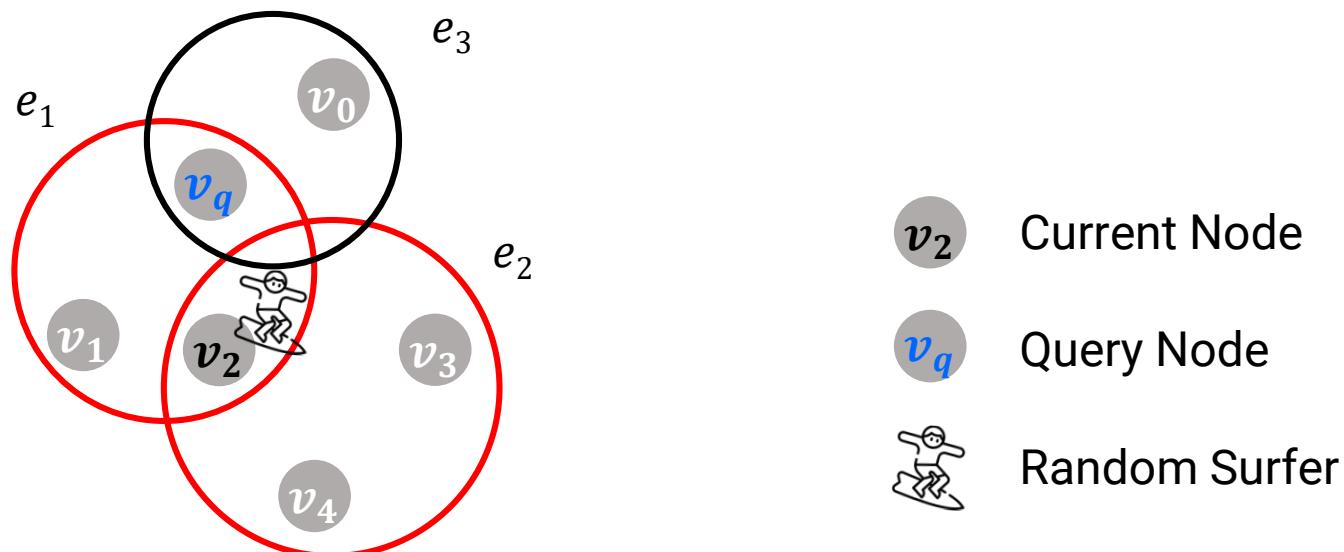
RWR on Hypergraphs: Definition

- Assume a random surfer who either..



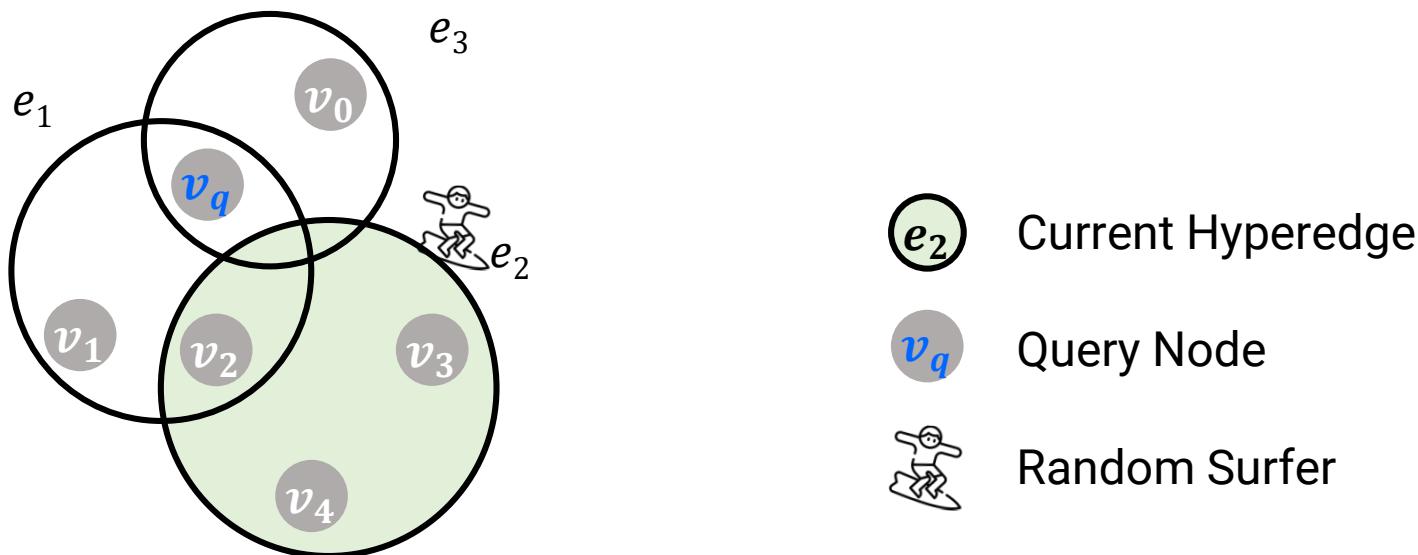
RWR on Hypergraphs: Definition (cont.)

- Assume a random surfer who either..
 - A1: Random walk (probability $1 - c$)
 - A1-1: Select a hyperedge e containing current node



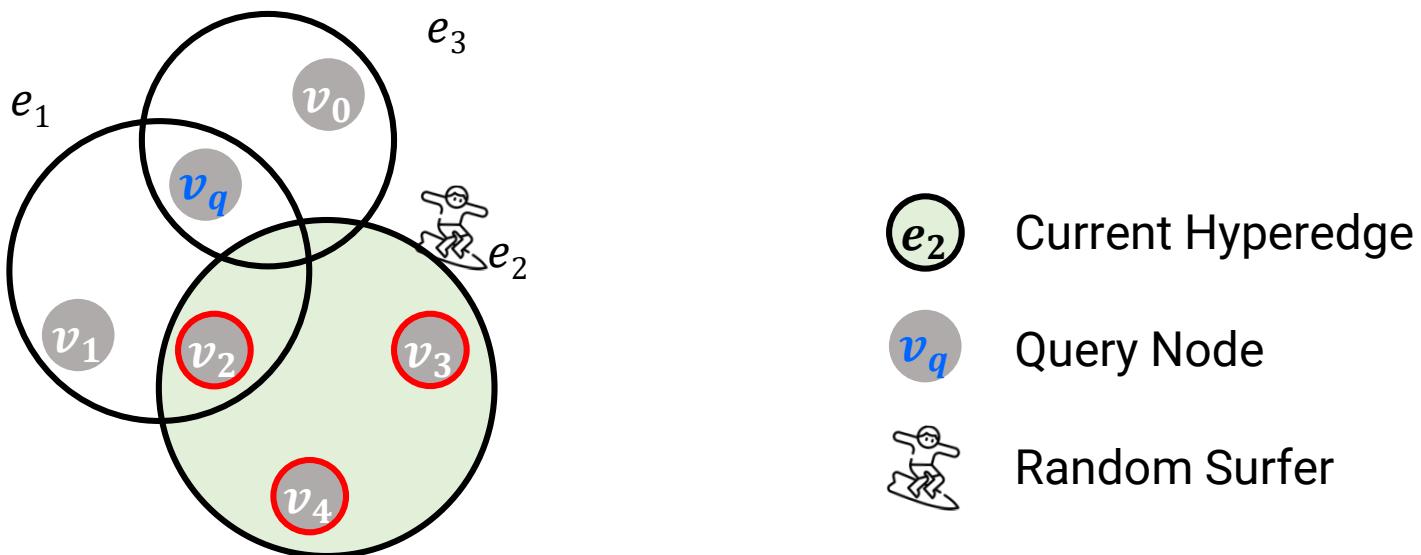
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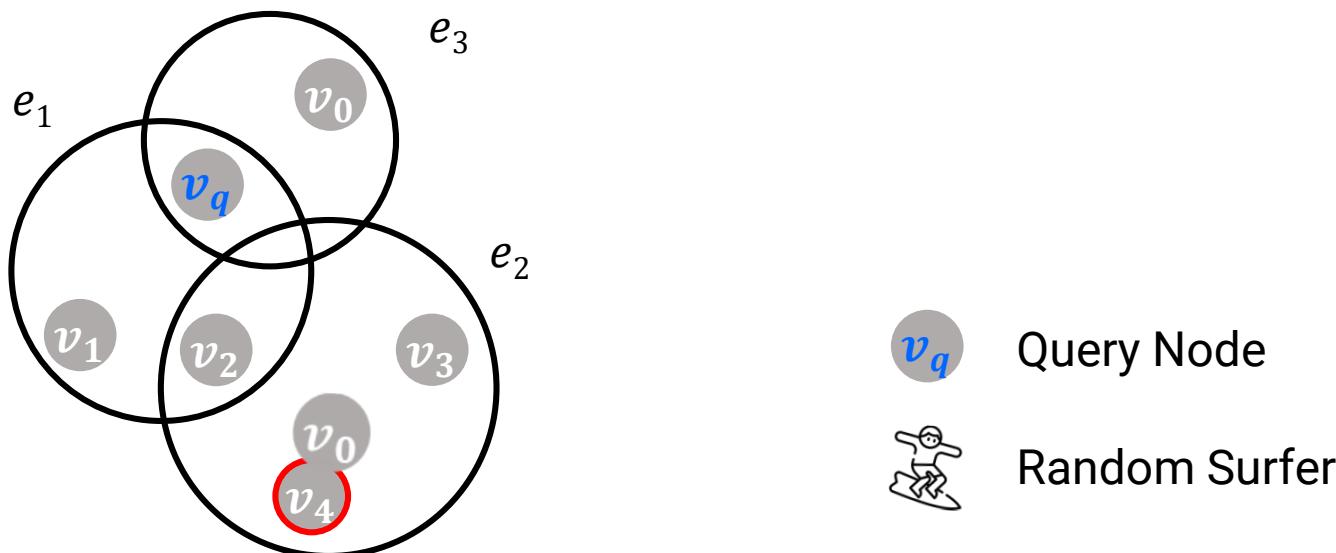
RWR on Hypergraphs: Definition (cont.)

- Assume a random surfer who either..
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 - A1-2: Move to a node v selected from the hyperedge e



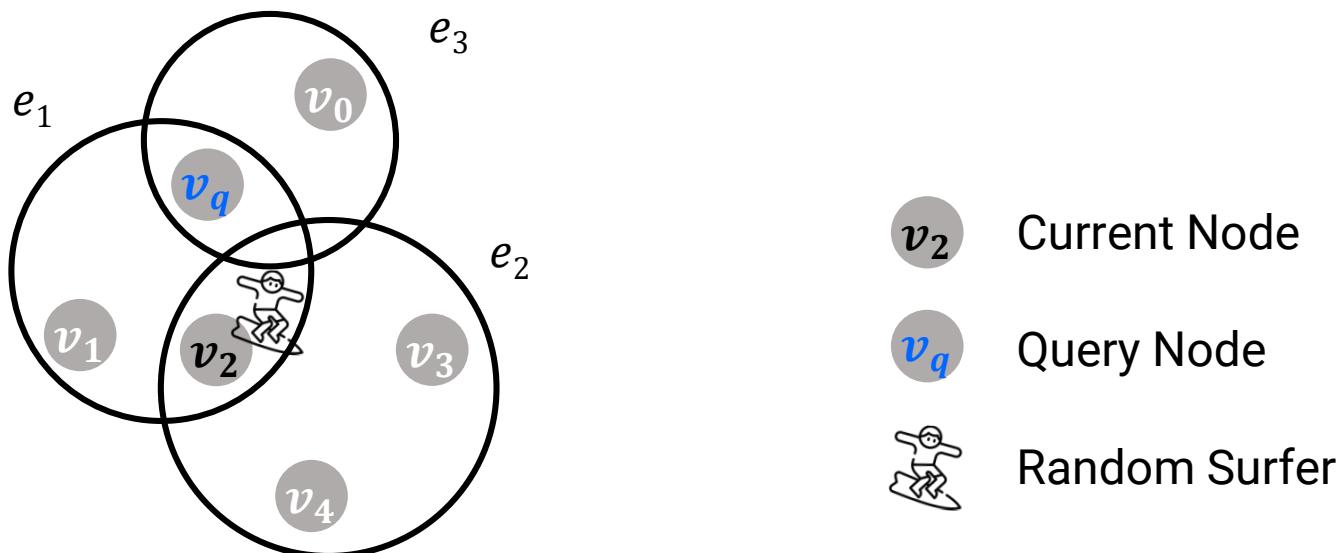
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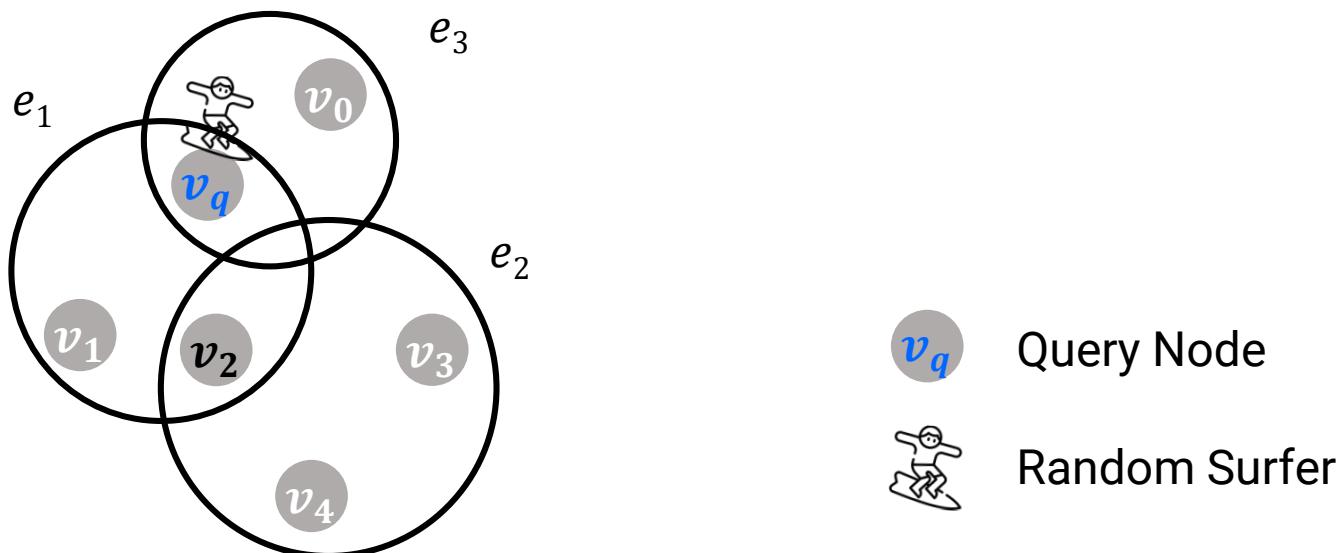
RWR on Hypergraphs: Definition (cont.)

- Assume a random surfer who either..
 - A1: Random walk (probability $1 - c$)
 - A2: Restart (probability c)
Restart at the query node



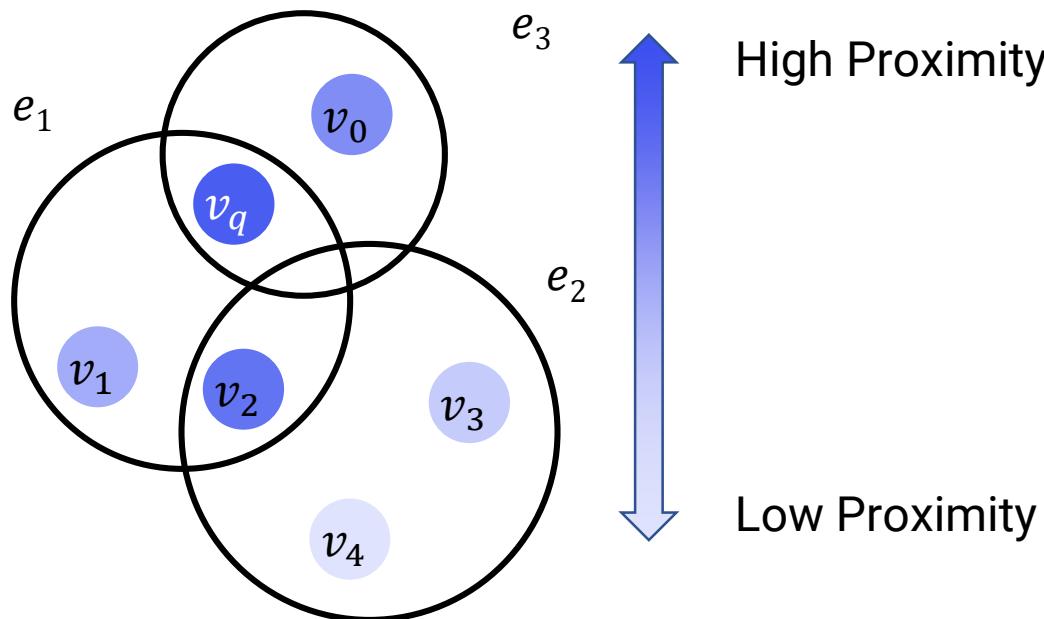
RWR on Hypergraphs: Definition (cont.)

- Assume a random surfer who either..
 - A1: Random walk (probability $1 - c$)
 - A2: Restart (**probability c**)
Restart at the query node



RWR on Hypergraphs: Definition (cont.)

- RWR score is defined as the **stationary probability** over the nodes
- RWR score is interpreted as the proximity between nodes

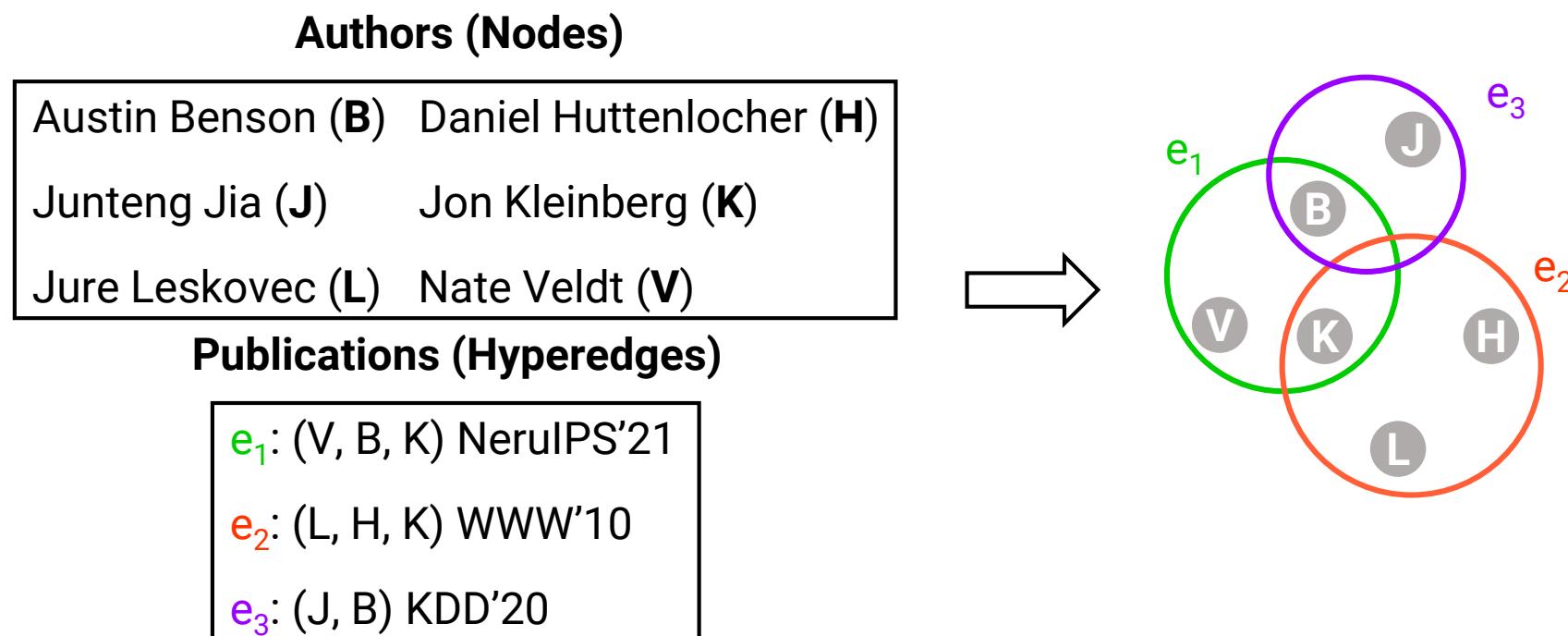


Node	Probability
v_q	0.34
v_2	0.22
v_0	0.19
v_1	0.14
v_3	0.07
v_4	0.04

RWR on Hypergraphs: Challenges

- In many real-world scenarios, RWR scores need to be calculated for **multiple queries**

co-authorship example



RWR on Hypergraphs: Challenges (cont.)

- In many real-world scenarios, RWR scores need to be calculated for **multiple queries**

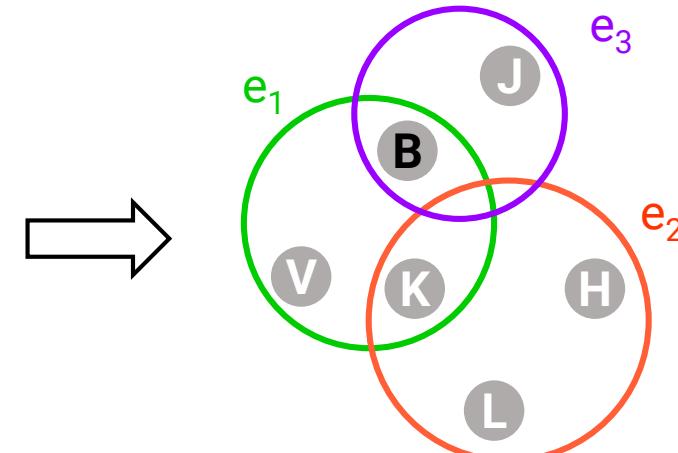
Which author is closest to **B**?

Authors (Nodes)

Austin Benson (B) Daniel Huttenlocher (H)
Junteng Jia (J) Jon Kleinberg (K)
Jure Leskovec (L) Nate Veldt (V)

Publications (Hyperedges)

e_1 : (V, B, K) NerulPS'21
 e_2 : (L, H, K) WWW'10
 e_3 : (J, B) KDD'20



RWR on Hypergraphs: Challenges (cont.)

- In many real-world scenarios, RWR scores need to be calculated for **multiple queries**

Which author is closest to K?

Authors (Nodes)

Austin Benson (B) Daniel Huttenlocher (H)

Junteng Jia (J) Jon Kleinberg (K)

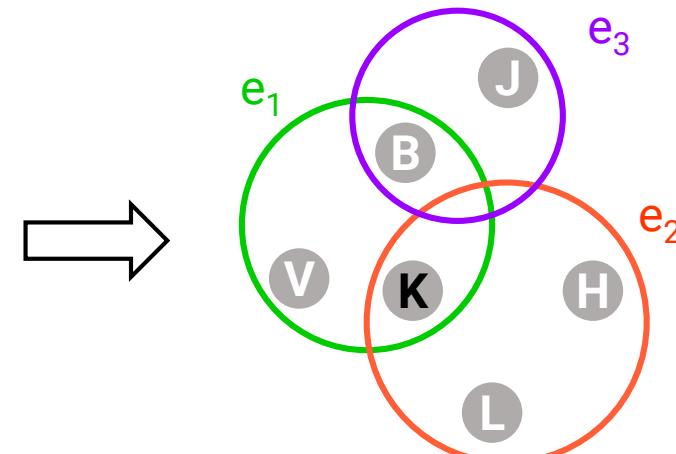
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RWR on Hypergraphs: Challenges (cont.)

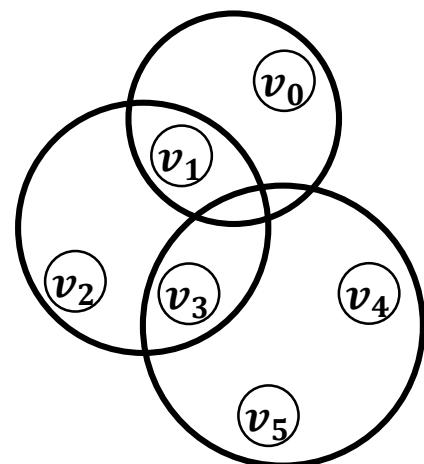
- In many real-world scenarios, RWR scores need to be calculated for **multiple queries**
- Q: Can we reduce the query cost?

RWR on Hypergraphs: Challenges (cont.)

- In many real-world scenarios, RWR scores need to be calculated for **multiple queries**
- Q: Can we reduce the query cost?
- A: **Preprocessing method!**
 - Preprocessing methods for **graphs**
 - BEAR (SIGMOD'15)
 - BePI (SIGMOD'17)

Preprocessing Method

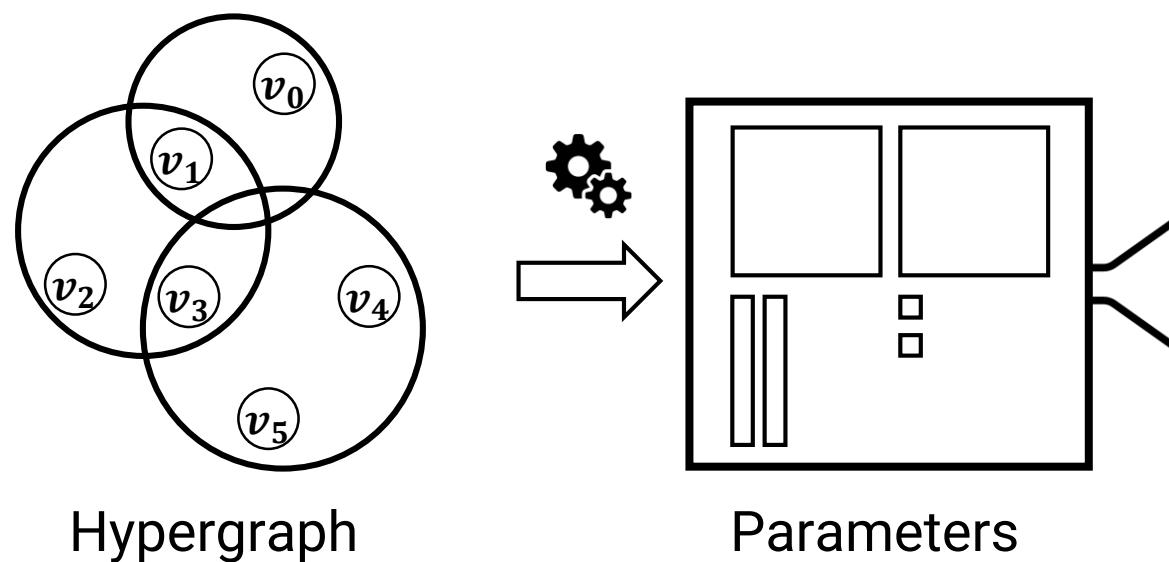
- Preprocess a given **hypergraph** into **parameters** for RWR
 - At the expense of preprocessing cost, lower the query cost



Hypergraph

Preprocessing Method (cont.)

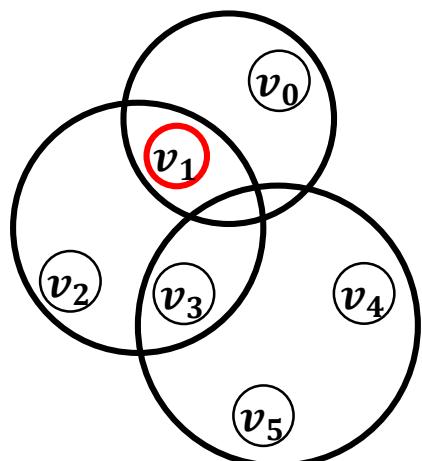
- Preprocess a given **hypergraph** into **parameters** for RWR
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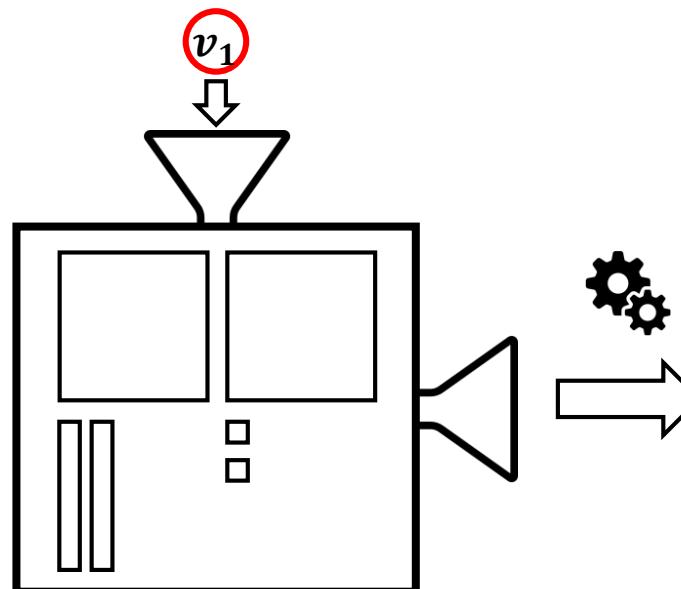
Preprocessing Method (cont.)

- Preprocess a given **hypergraph** into **parameters** for RWR
 - At the expense of preprocessing cost, lower the query cost

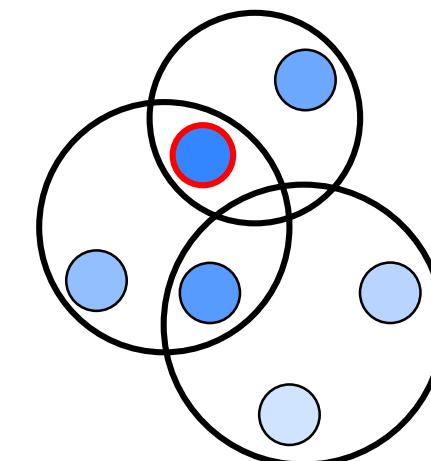
What is the proximity of nodes w.r.t. v_1 ?



Hypergraph



Parameters

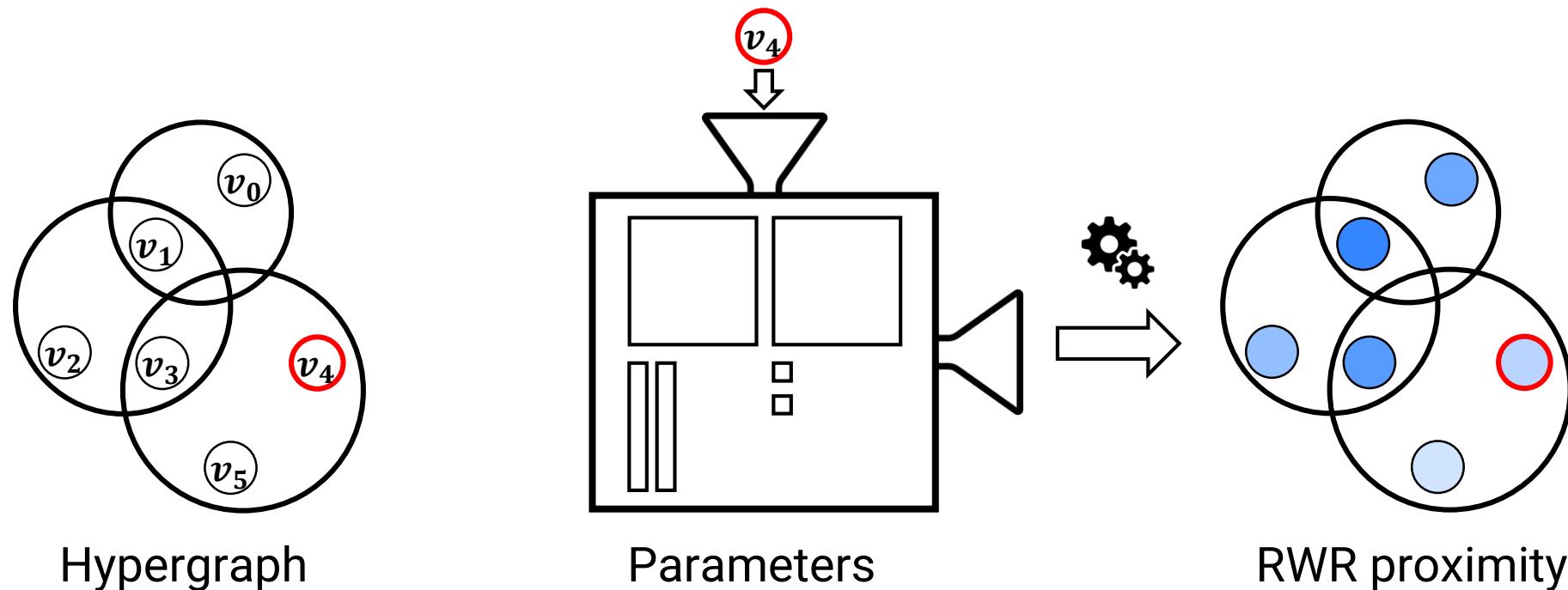


RWR proximity

Preprocessing Method (cont.)

- Preprocess a given **hypergraph** into **parameters** for RWR
 - At the expense of preprocessing cost, lower the query cost

What is the proximity of nodes w.r.t. v_4 ?



Preprocessing Method (cont.)

- Preprocess a given **hypergraph** into **parameters** for RWR
 - At the expense of preprocessing cost, lower the query cost
- There is no preprocessing method for RWR on **hypergraphs**

Our Goal

- Develop a **fast preprocessing method for RWR on Hypergraphs**

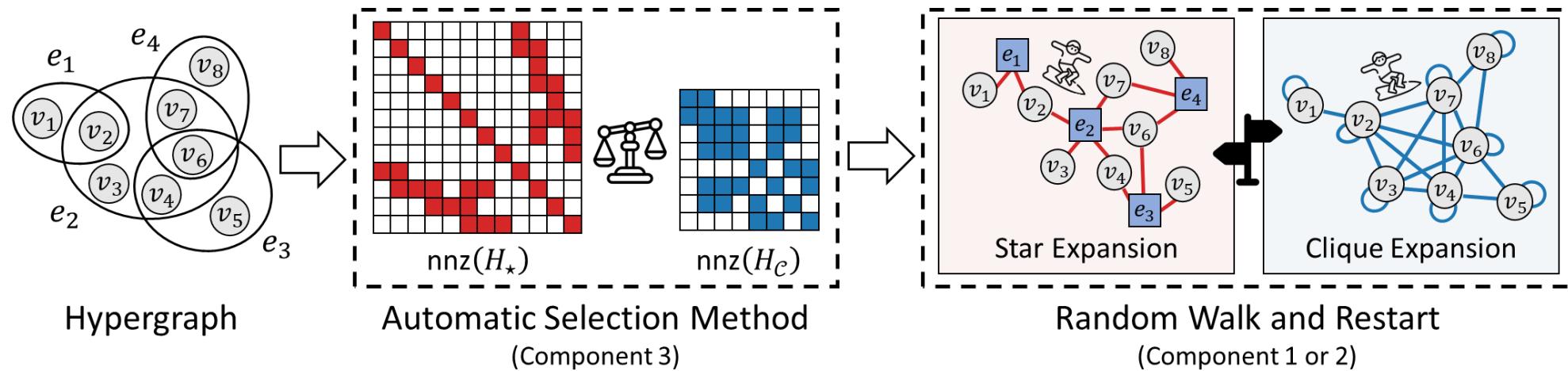
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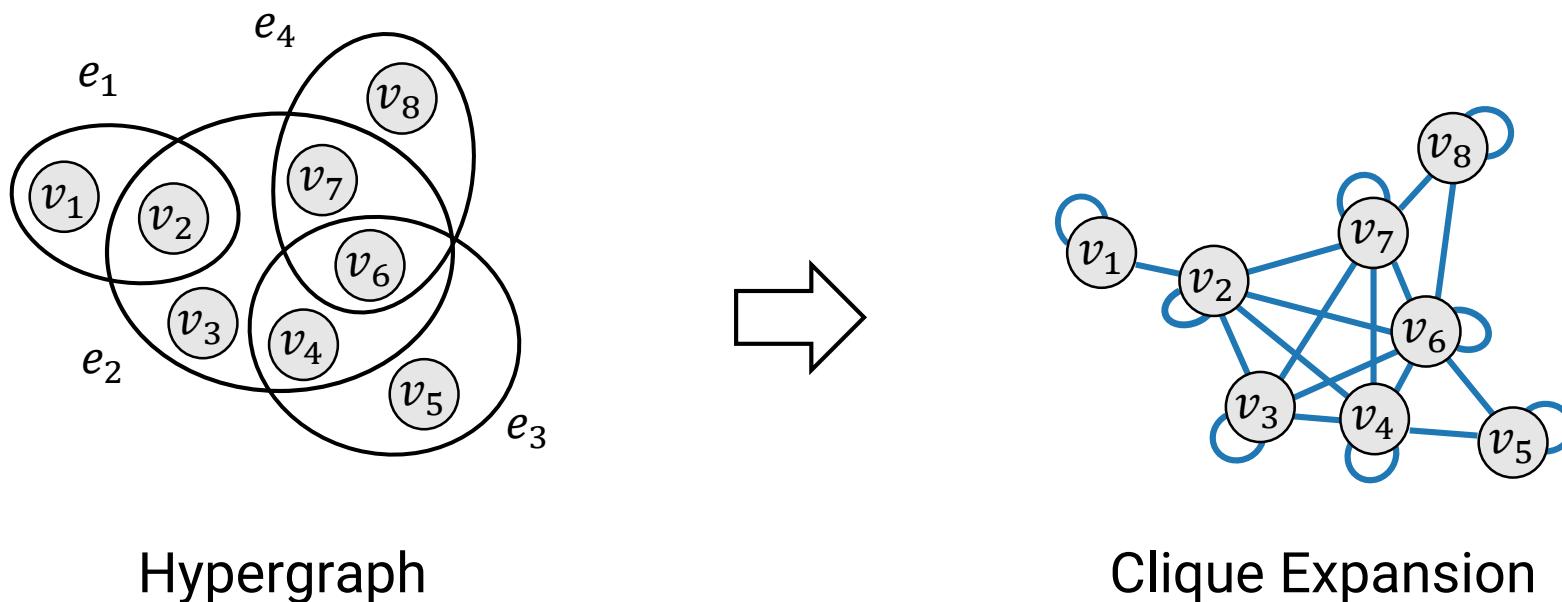
Proposed Algorithm: ARCHER

- We propose **ARCHER** (Adaptive RWR Computation on Hypergraphs)
- Component 1: Clique-expansion-based Method
- Component 2: Star-expansion-based Method
- Component 3: Automatic Selection Method



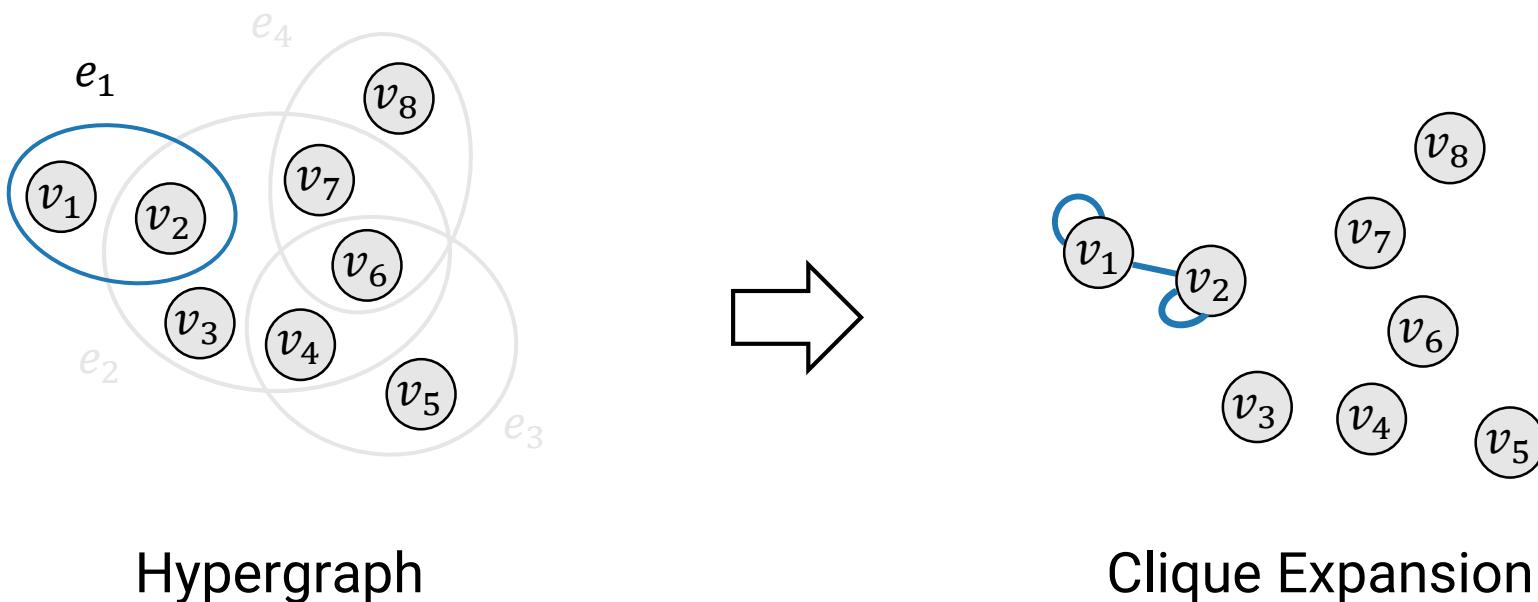
Comp. 1: Clique-expansion-based method

- Clique-expansion: a graph constructed from a hypergraph by replacing each original hyperedge with a clique



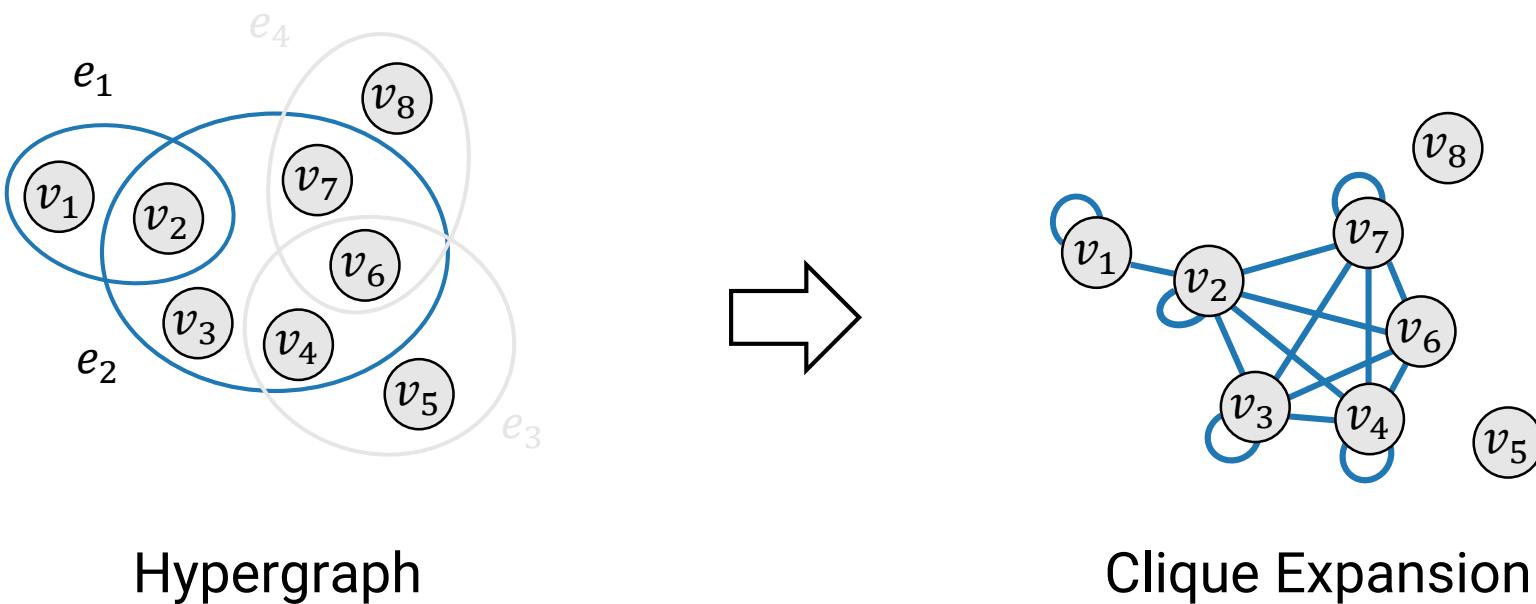
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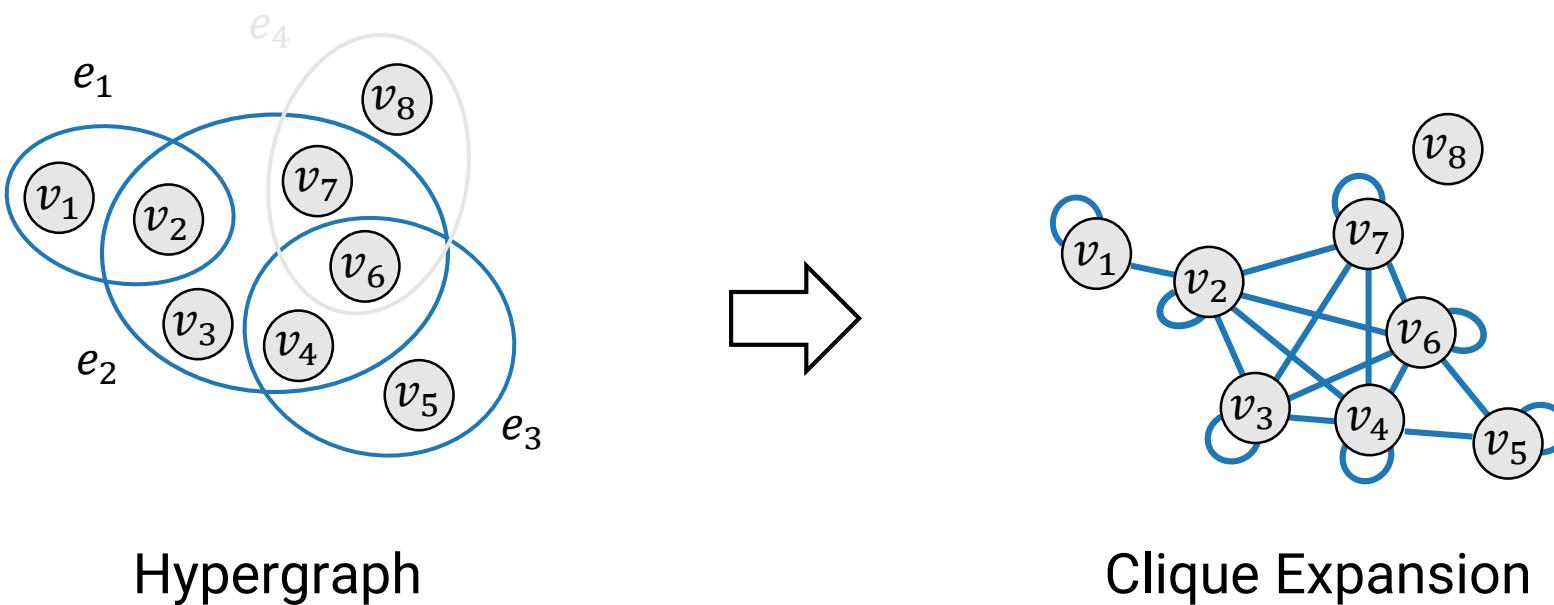
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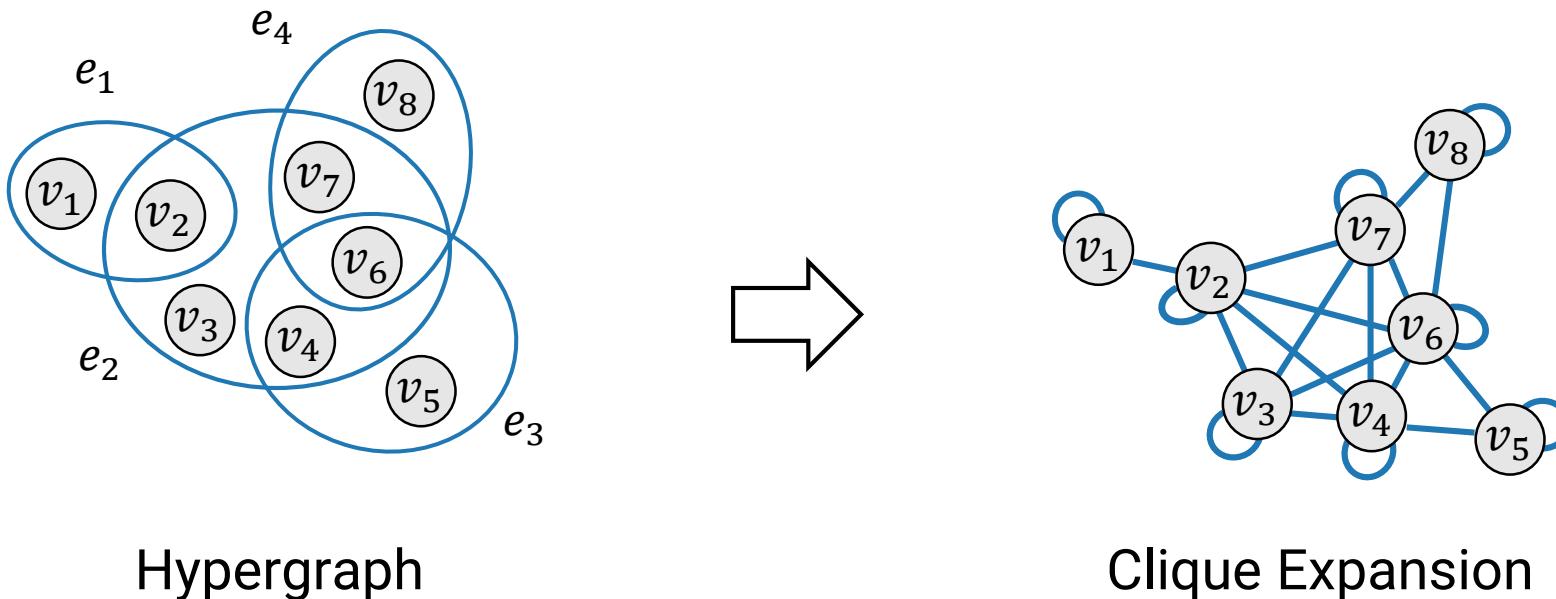
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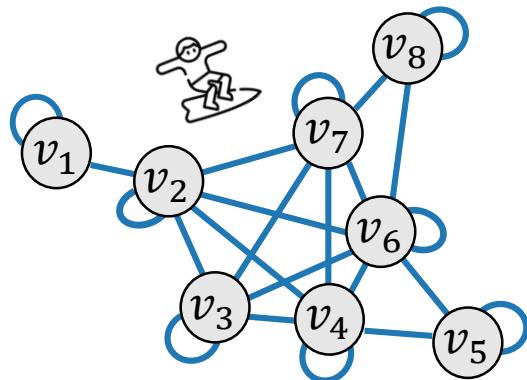
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Comp. 1: Clique-expansion-based method (cont.)

- **Theorem (Clique-Expansion Equality):** RWR on a hypergraph can be reduced to RWR on its clique-expanded graph with proper edge weights



Clique Expansion

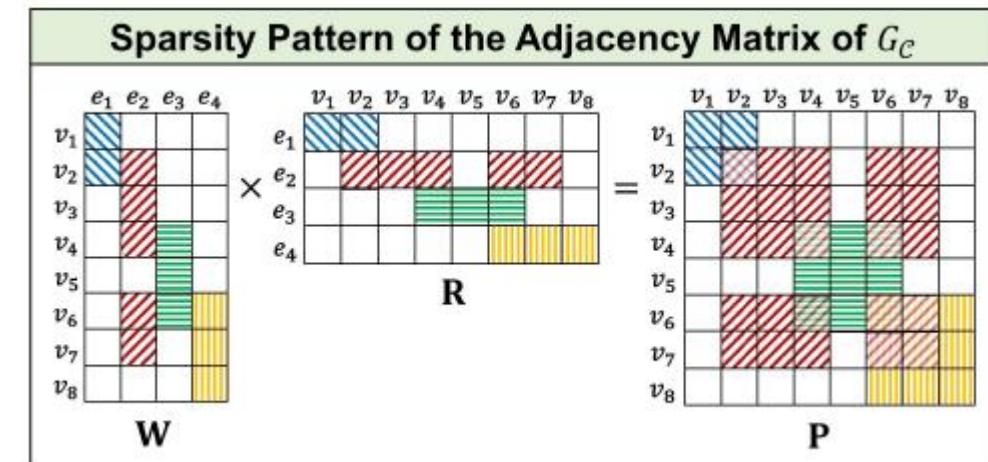
1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0
0	1	0	1	1	0	0	0
0	0	1	1	0	1	0	0
0	0	1	0	1	0	1	0
0	0	0	0	1	0	0	1
0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1

Adjacency Matrix of the Clique-Expanded Graph

Comp. 1: Clique-expansion-based method (cont.)

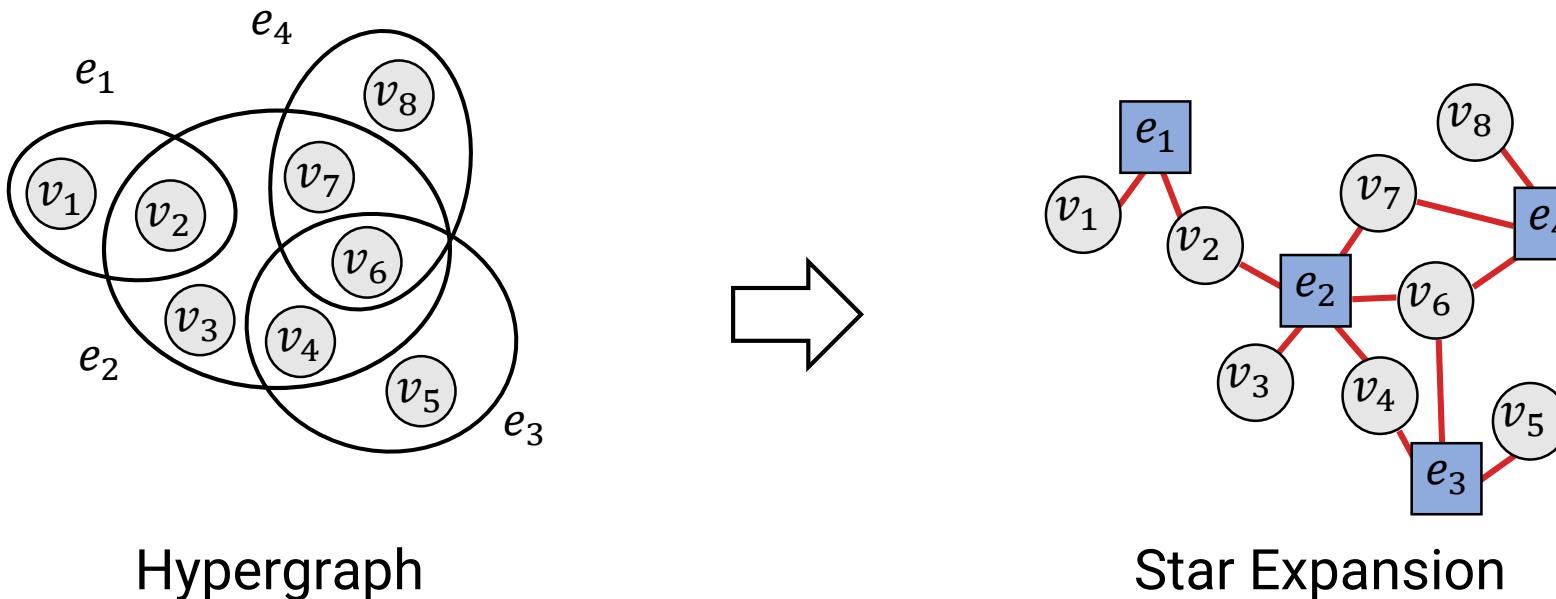
Details

- **Theorem (Clique-Expansion Equality):** RWR on a hypergraph can be reduced to RWR on its clique-expanded graph with proper edge weights
- Input
 - Row-normalized hyperedge-weight matrix \widetilde{W}
 - Row-normalized node-weight matrix \widetilde{R}
 - RWR query vector q
 - Restart probability c
- RWR score vector r can be obtained by solving
 - $r = (1 - c)(\widetilde{W}\widetilde{R})^\top r + cq$



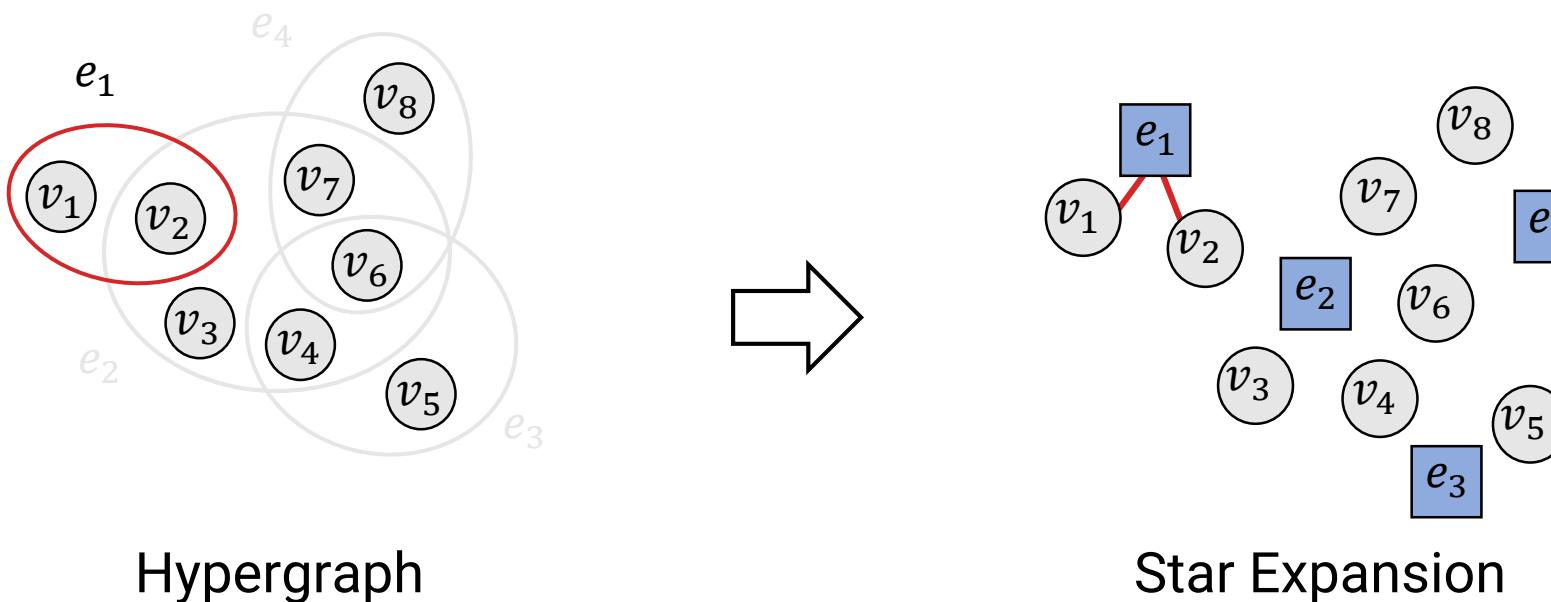
Comp. 2: Star-expansion-based method

- Star-expansion: a graph constructed from a hypergraph by
 - (1) aggregating nodes and hyperedges into new set of nodes
 - (2) adding edges between each incident node and hyperedge pair



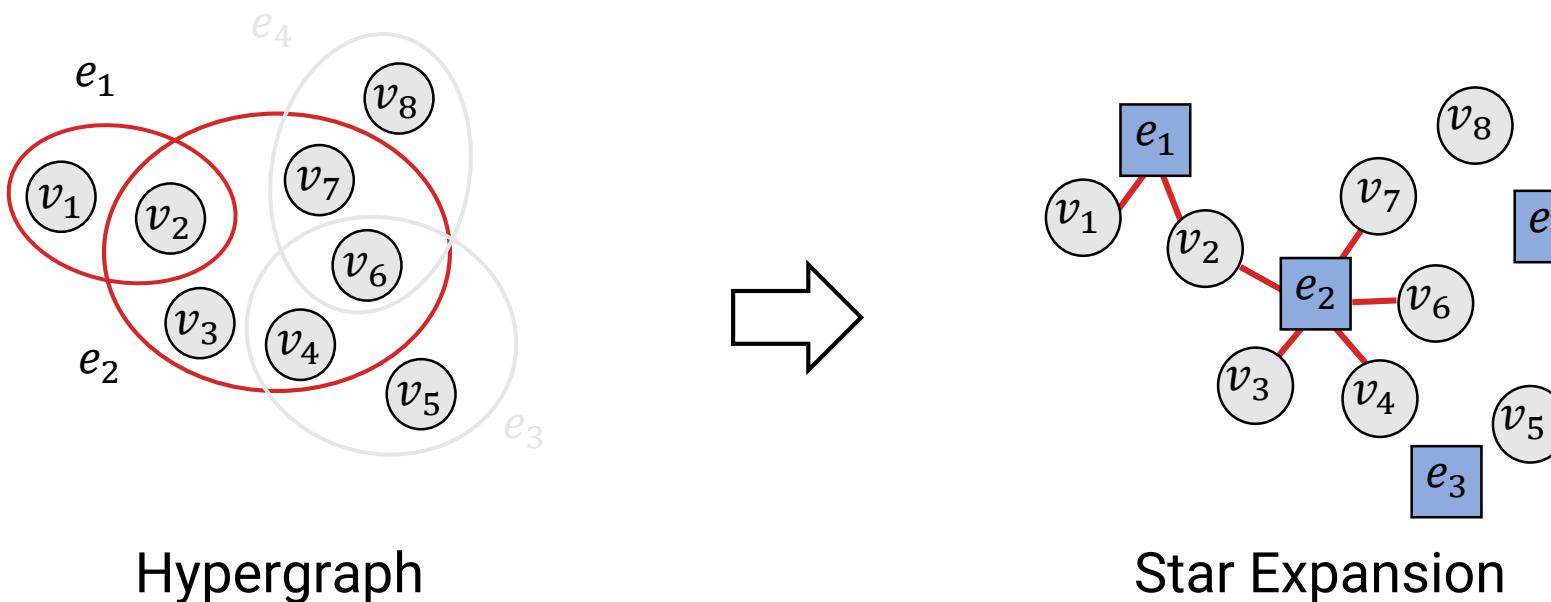
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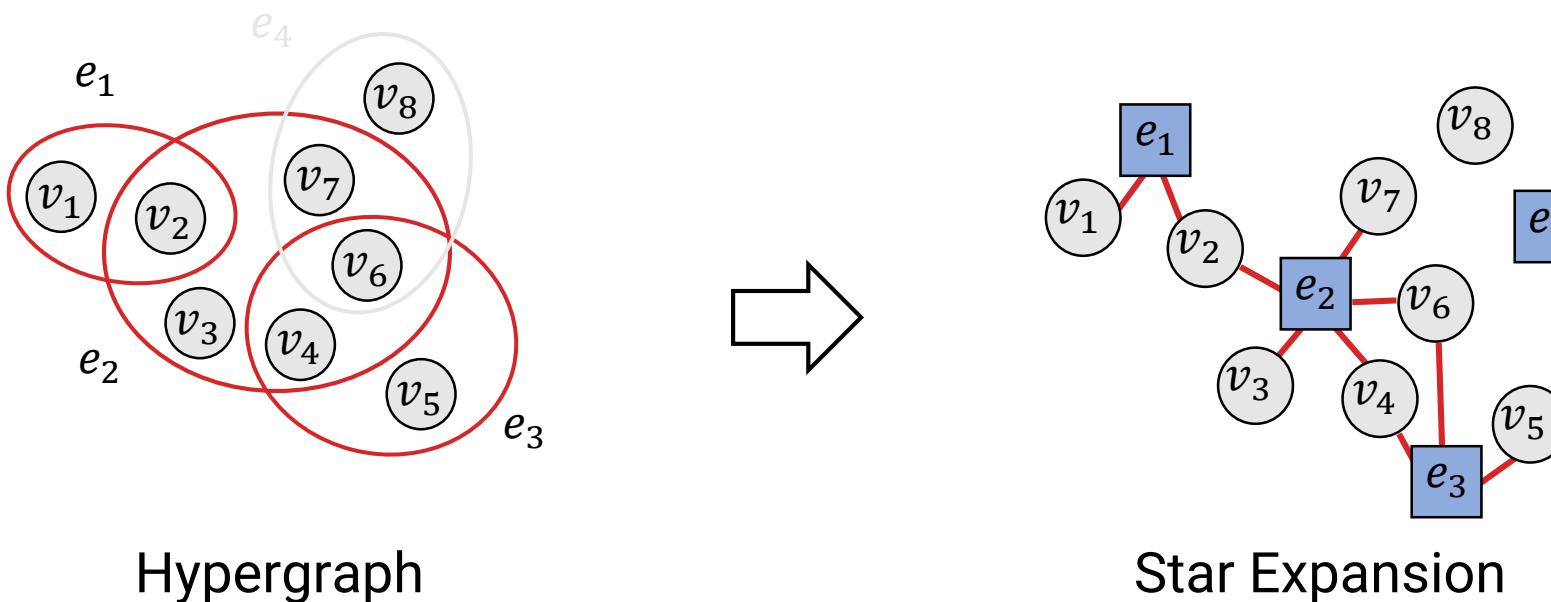
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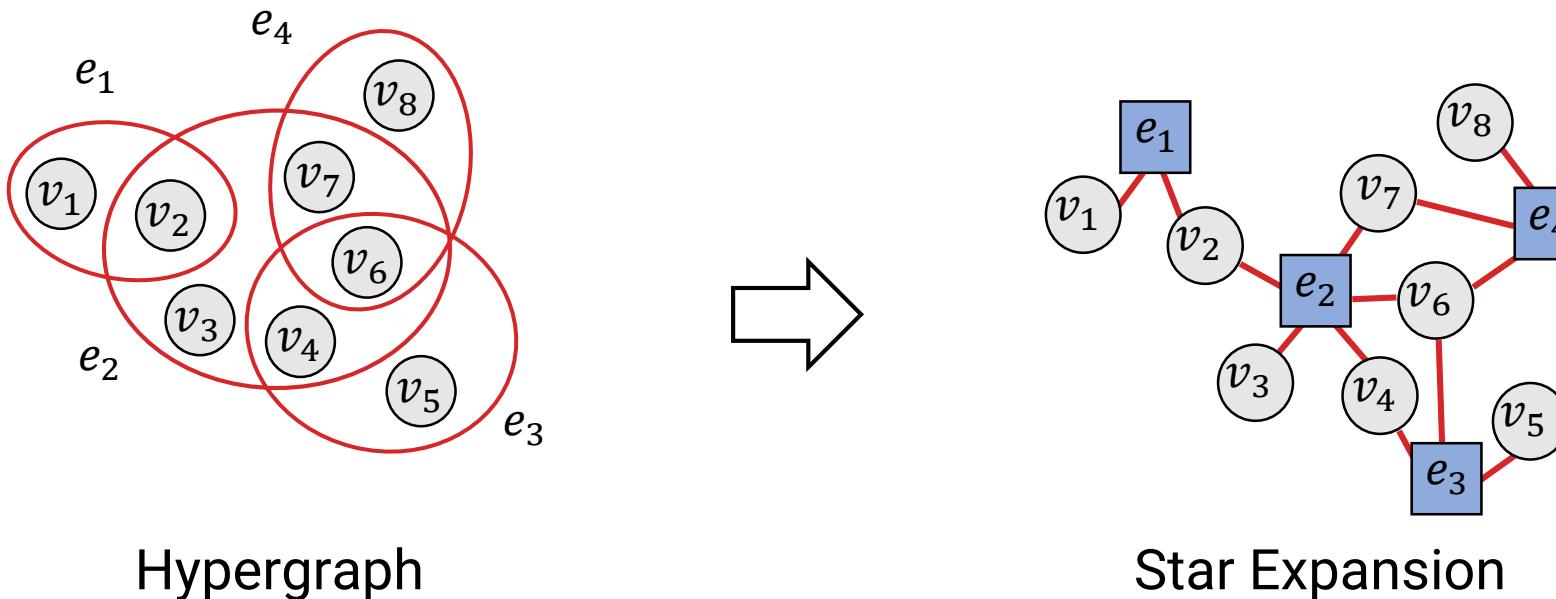
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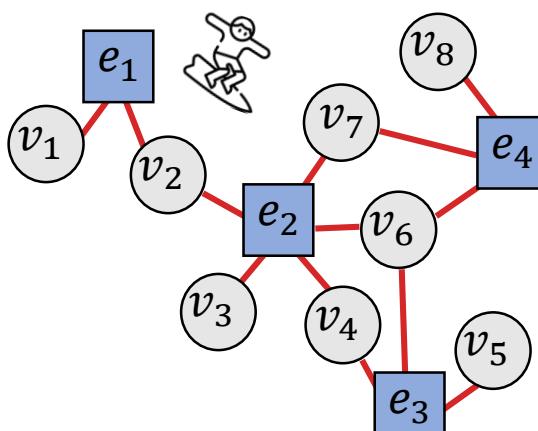
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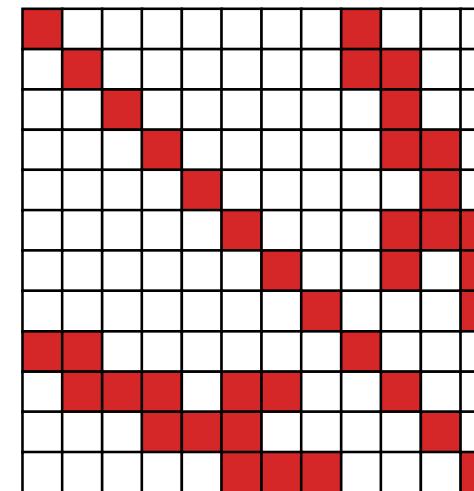


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- **Theorem (Star-Expansion Equality):** RWR on a hypergraph can be reduced to RWR on its star-expanded graph with proper edge weights



Star Expansion

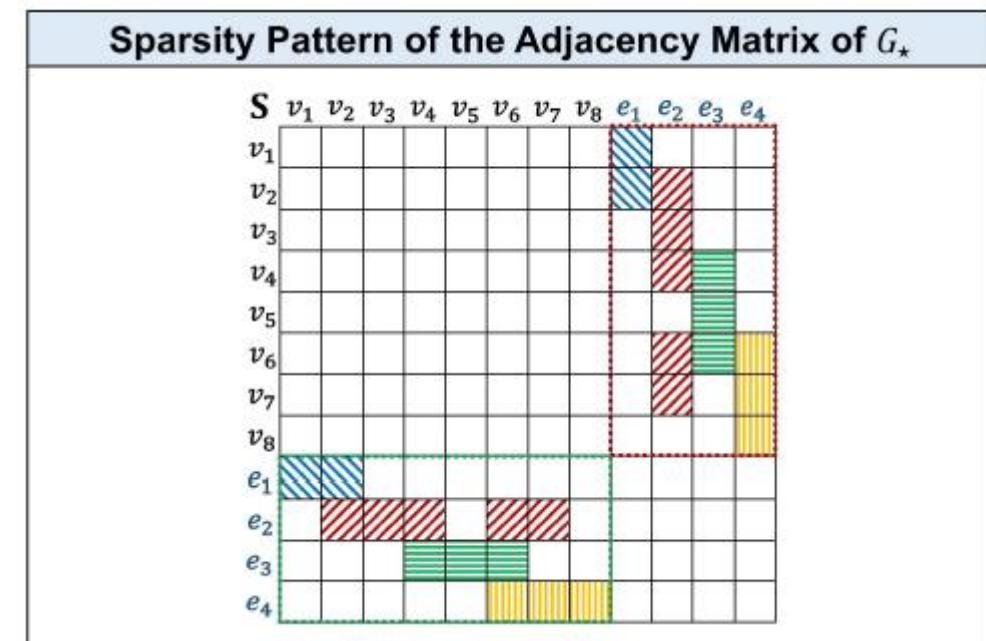


Adjacency Matrix of the
Star-Expanded Graph

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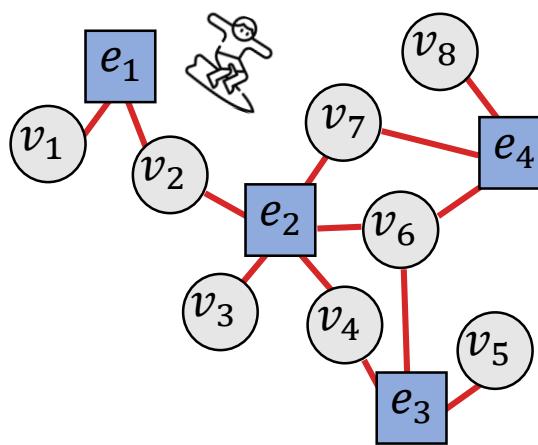
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 - Row-normalized node-weight matrix \widetilde{R}
 - RWR query vector q
 - Restart probability c
- RWR score vector r can be obtained by solving
 - $r_* = (1 - c_*) \left(\begin{bmatrix} \mathbf{0} & \widetilde{W} \\ \widetilde{R} & \mathbf{0} \end{bmatrix} \right)^T r_* + c_* \begin{bmatrix} q \end{bmatrix},$
 - $r = \frac{c}{c_*} r_*[1:|V|] \quad (c_* = 1 - \sqrt{1 - c})$

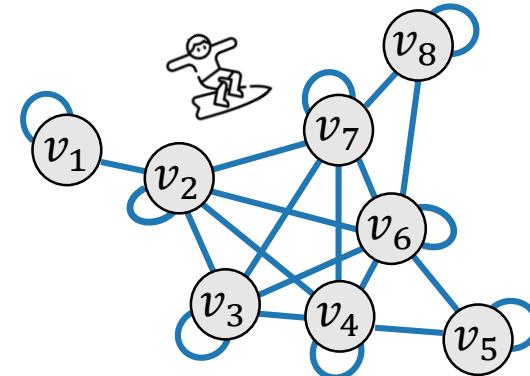


Comp. 3: Automatic selection method

- How can we choose between clique- and star-expansion?



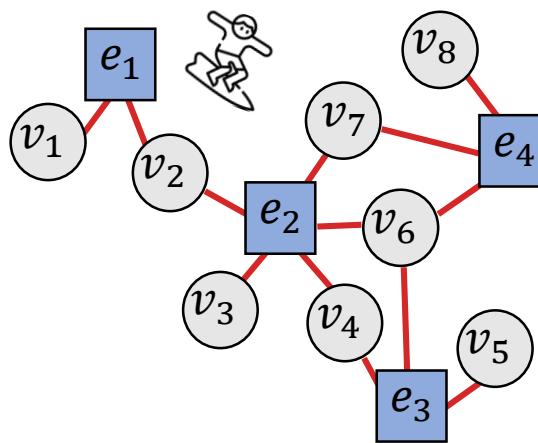
Star Expansion



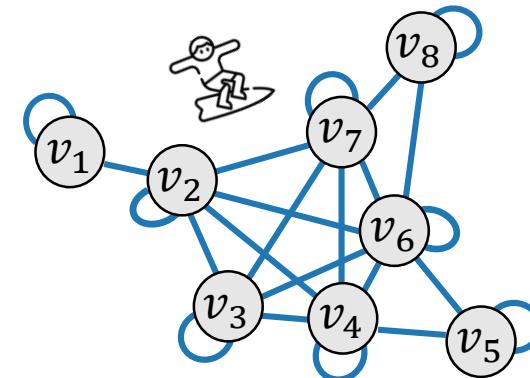
Clique Expansion

Comp. 3: Automatic selection method (cont.)

- How can we choose between clique- and star-expansion?
- Naïve answer: preprocess both, choose after running



Star Expansion



Clique Expansion

Comp. 3: Automatic selection method (cont.)

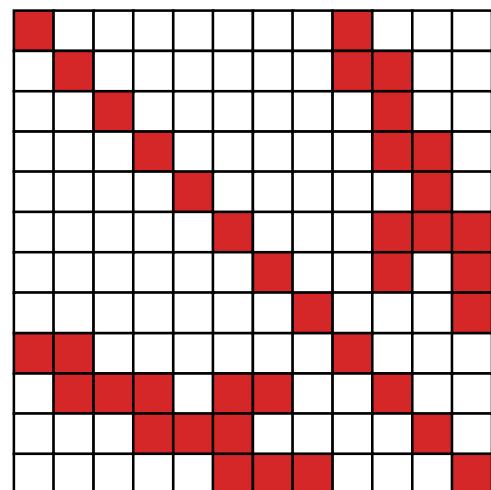
- How can we choose between clique- and star-expansion?
- Naïve answer: preprocess both, choose after running
- Cost of preprocessing vary depending on the dataset
 - Empirical results
 - Case 1: star-expansion-based method is efficient
 - Up to 137.6x less time
 - Up to 16.2x less space
 - Case 2: clique-expansion-based method is efficient
 - Up to 6.4x less time
 - Up to 9.6x less space

*Is there a way to choose **without actually executing them**?*

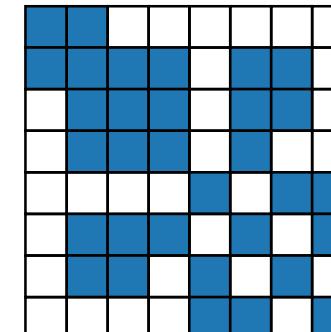
Comp. 3: Automatic selection method (cont.)

- *Hint:* empirically preprocessing cost depends on **the count of non-zeros in the adjacency matrix**

Star Expansion



Clique Expansion



Comp. 3: Automatic selection method (cont.)

- Computing the **non-zero count** is much lighter than preprocessing!
- Computing cost of **counting non-zeros**
 - Star expansion
 - Time complexity: $O(1)$
 - Space complexity: $O(1)$
 - Clique expansion
 - Time complexity: $O(\sum_{e \in E} |e|^2)$
 - Space complexity: $O(n)$

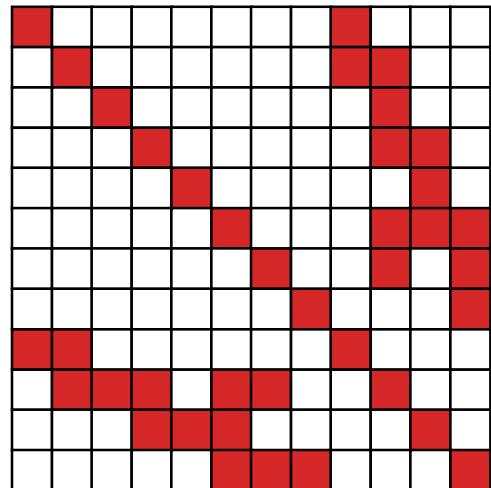
※Worst case preprocessing cost※

- Star expansion
 - Time: $O((|V| + |E|)^3)$
 - Space: $O((|V| + |E|)^2)$
- Clique expansion
 - Time: $O(|V|^3)$
 - Space: $O(|V|^2)$

Comp. 3: Automatic selection method (cont.)

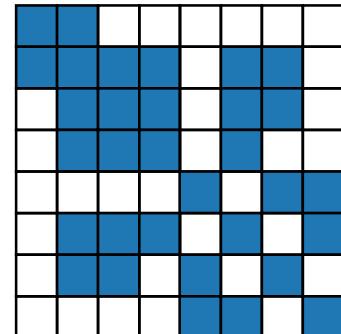
- Choose the method with **less non-zeros**

Star Expansion



`nonzero(star)`

Clique Expansion

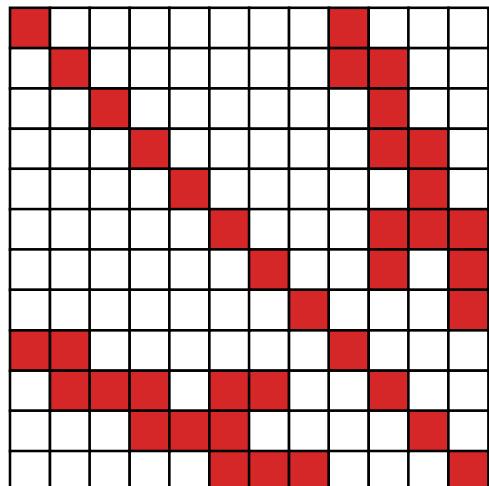


`nonzero(clique)`

Comp. 3: Automatic selection method (cont.)

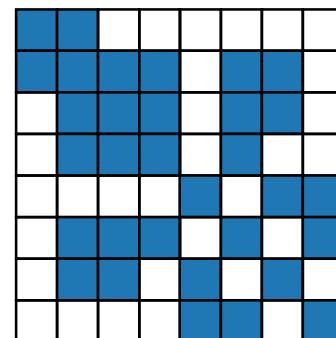
- Choose the method with **less non-zeros**

Star Expansion

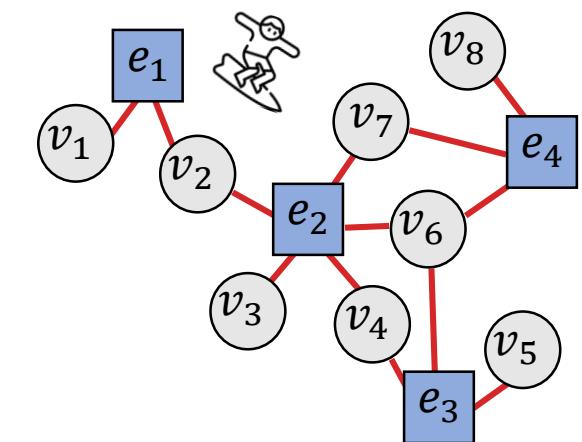
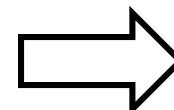


$\text{nonzero}(\text{star}) <$

Clique Expansion



$\text{nonzero}(\text{clique})$

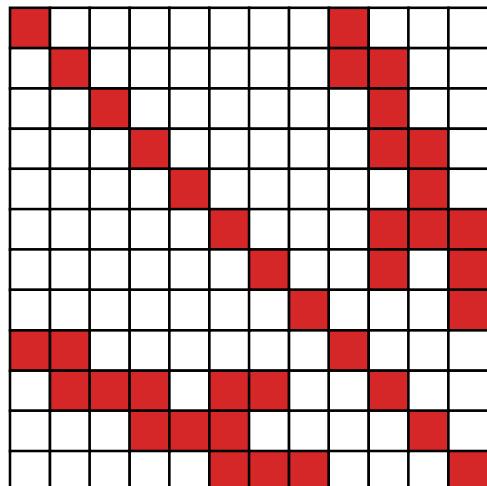


Star Expansion

Comp. 3: Automatic selection method (cont.)

- Choose the method with **less non-zeros**

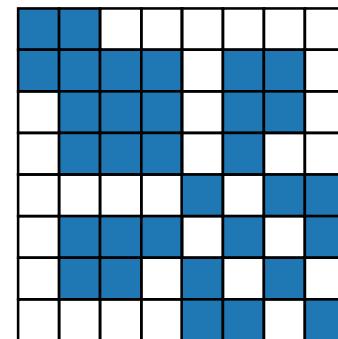
Star Expansion



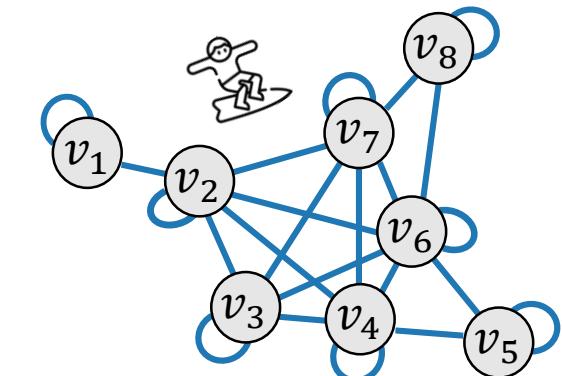
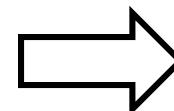
$\text{nonzero}(\text{star})$

$>$

Clique Expansion

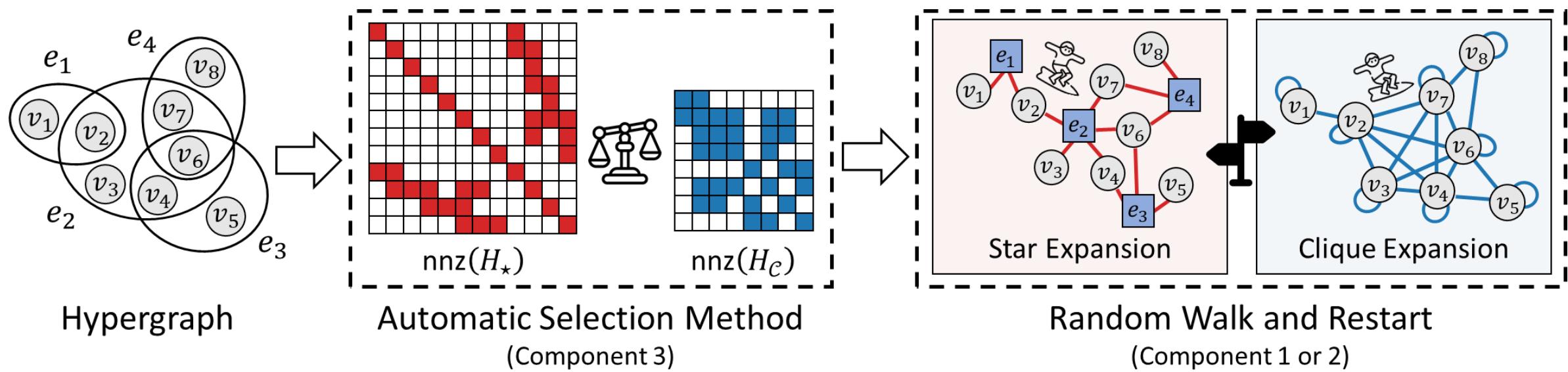


$\text{nonzero}(\text{clique})$



Clique Expansion

Ultimate Framework: ARCHER



Roadmap

- Overview
- Random Walk with Restart (RWR) on Hypergraphs
- Proposed Algorithm: ARCHER
- [Empirical Evaluation of ARCHER <<](#)
- Application: Anomaly Detection
- Conclusion



Experimental Settings

- Datasets: 18 real-world hypergraphs from 10 different domains
 - # nodes: 143 – 2.7M
 - # hyperedges: 6,038 – 11M

Dataset	n	m	$\text{avg}_{e \in \mathcal{E}} e $	$\text{max}_{e \in \mathcal{E}} e $	Density	Overlapness
EEN	143	10,883	2.47	37	76.12	188.21
EEU	998	234,760	2.39	40	234.09	559.80
SB	294	29,157	7.96	99	99.17	789.62
HB	1,494	60,987	20.47	399	40.82	835.79
WAL	88,860	69,906	6.59	25	0.79	5.81
TRI	172,738	233,202	3.12	85	1.35	4.21
AM	55,700	105,655	8.12	555	1.90	15.41
YP	25,252	25,656	18.2	649	1.02	18.50
TW	81,305	70,097	25.2	1,205	0.86	21.75
COH	1,014,734	1,812,511	1.32	925	1.75	2.32
COG	1,256,385	1,590,335	2.80	284	1.26	3.53
COD	1,924,991	3,700,067	2.79	280	1.92	5.35
THU	125,602	192,947	1.80	14	1.54	2.76
THM	176,445	719,792	2.24	21	4.08	9.13
THS	2,675,955	11,305,343	2.23	67	4.22	9.56
ML1	3,533	6,038	95.3	1,435	1.71	162.83
ML10	10,472	69,816	84.3	3,375	6.67	562.02
ML20	22,884	138,362	88.1	4,168	6.05	532.93

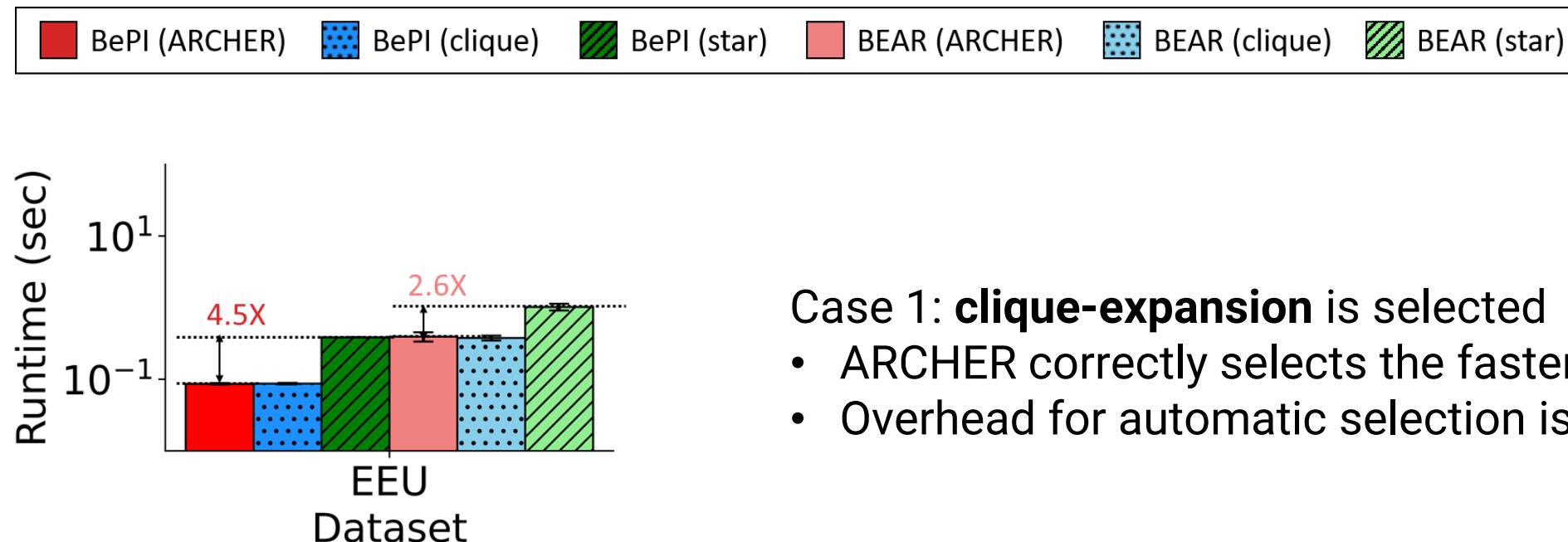
- **Email:** EEN, EEU
- **Co-sponsorship of bills:** SB, HB
- **Co-purchased products:** WAL
- **“Click-out” accommodation set:** TRI
- **Co-authorship:** COD, COG, COH
- **User group of thread:** THS, THM, THU
- **Product review:** AM
- **Location review:** YP
- **Group on Twitter:** TW
- **Movie review:** ML1, ML10, ML20

Experimental Settings (cont.)

- RWR computation methods for hypergraphs:
 - ARCHER
 - Always clique-expansion-based method
 - Always star-expansion-based method
- RWR computation methods for clique- and star-expanded graphs:
 - Preprocessing methods
 - BEAR (SIGMOD'15)
 - BePI (SIGMOD'17)
 - Power iteration

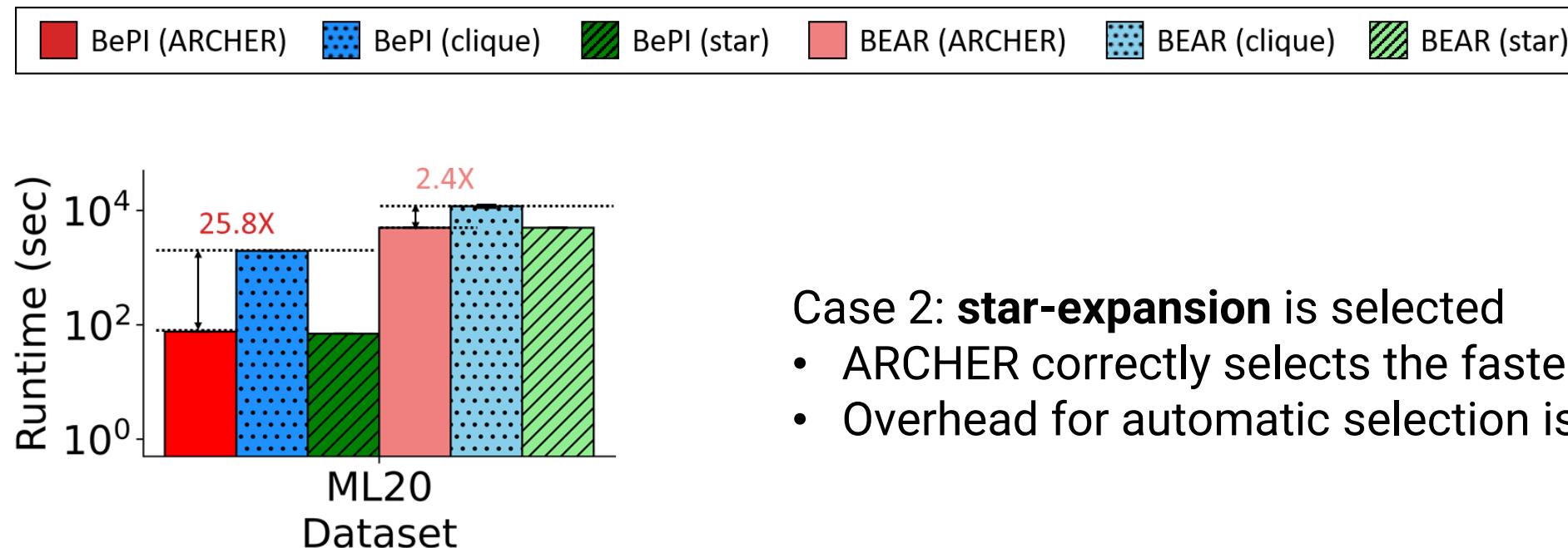
Q1. Preprocessing Time

- Q1. How **long** do ARCHER and the baselines take for preprocessing?



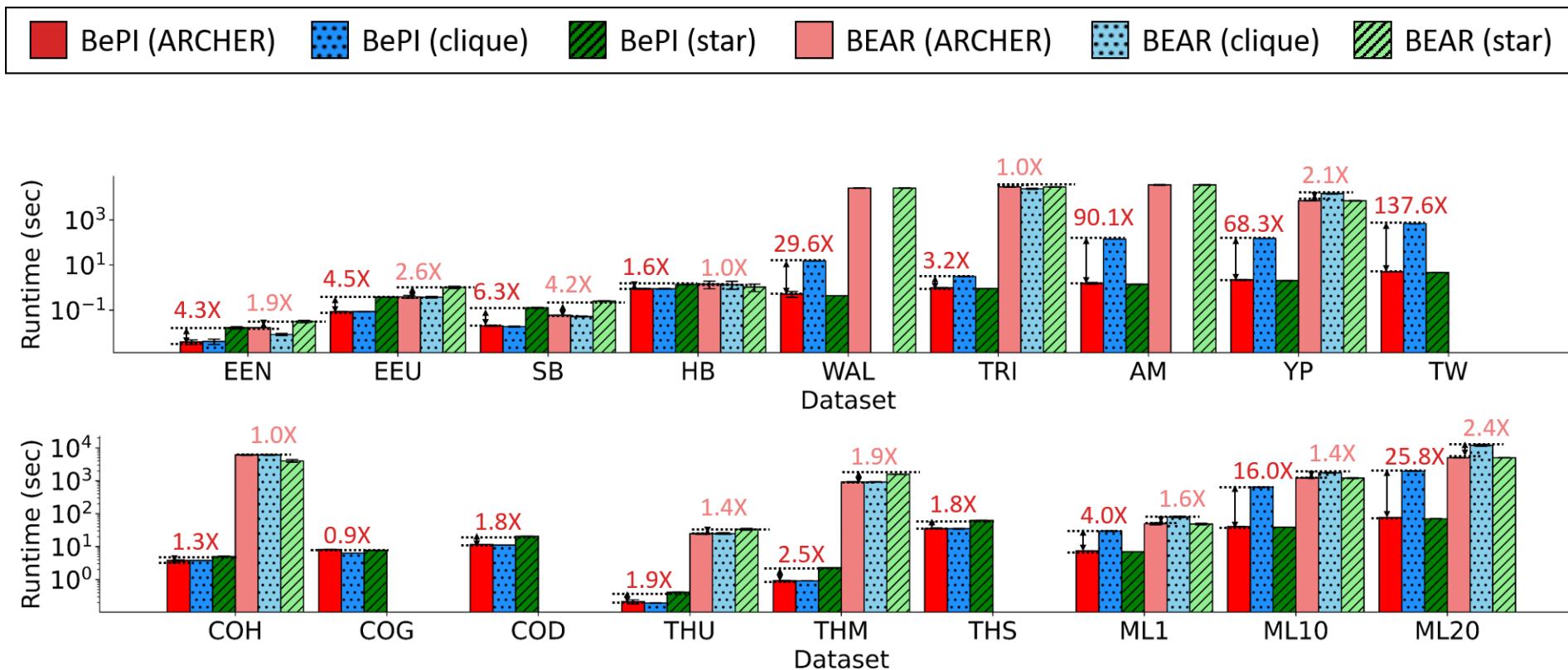
Q1. Preprocessing Time (cont.)

- Q1. How **long** do ARCHER and the baselines take for preprocessing?



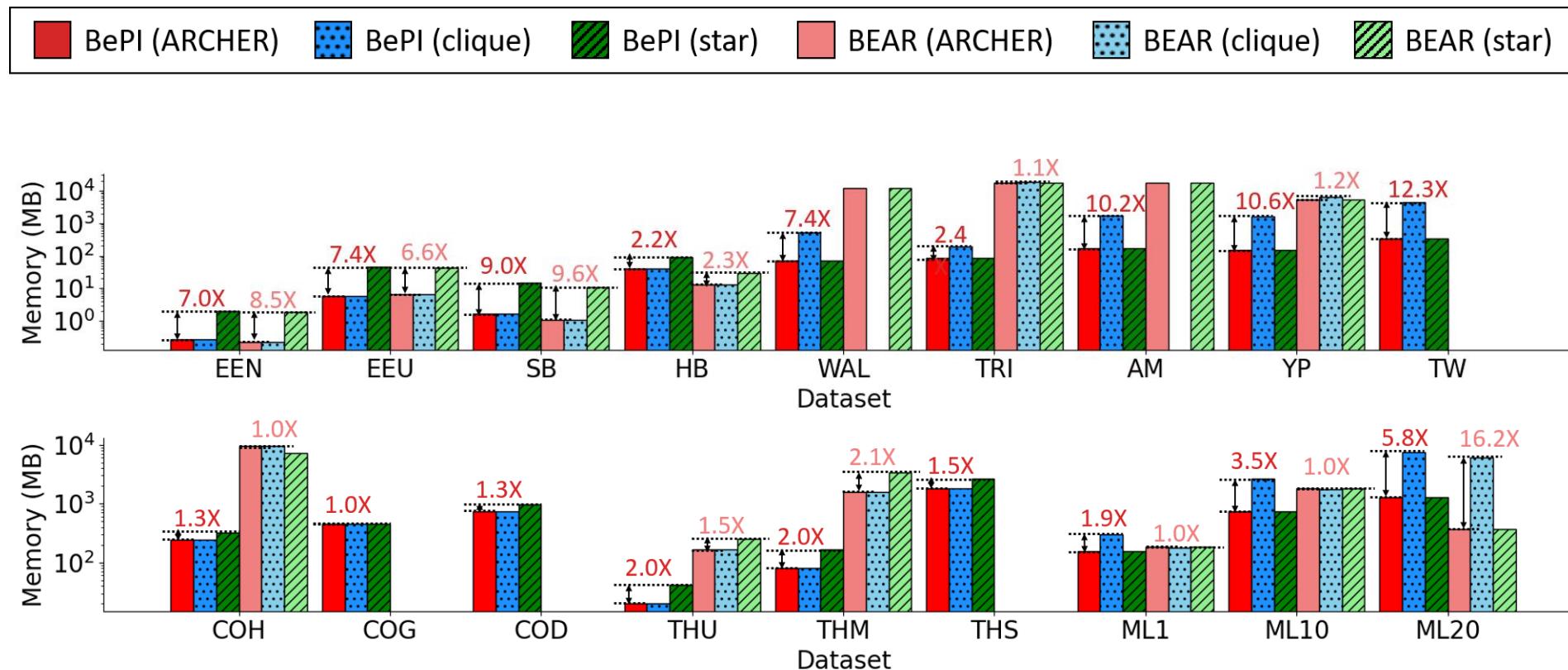
Q1. Preprocessing Time (cont.)

- Q1. How **long** do ARCHER and the baselines take for preprocessing?
- A1. ARCHER takes up to **137.6X** less time



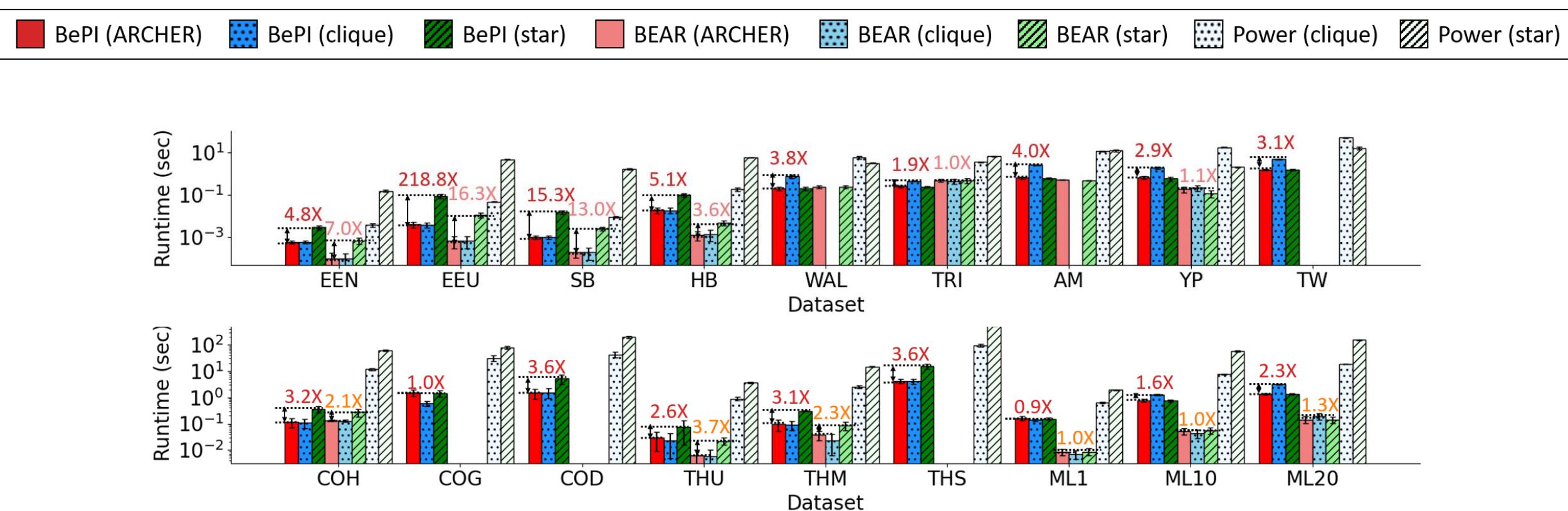
Q2. Space Cost

- Q2. How much **memory space** do they take for preprocessing?
- A2. ARCHER takes up to **16.2X** less space



Q3. Query Time

- Q3. How quickly do they process RWR queries?
- A3. ARCHER takes up to **218.8x** less time

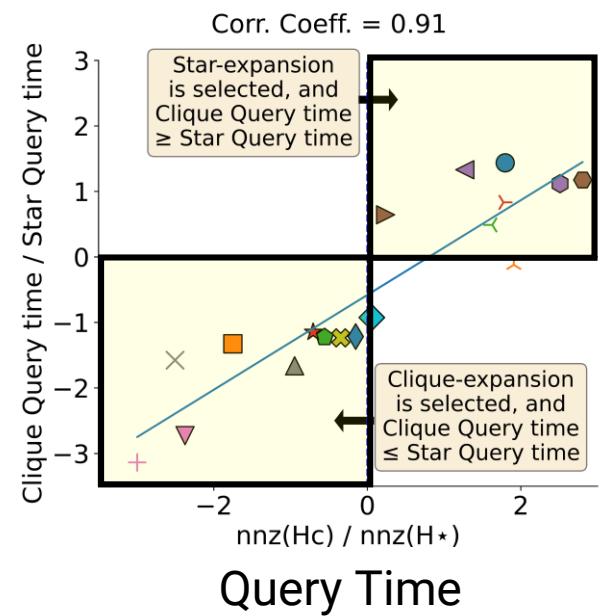
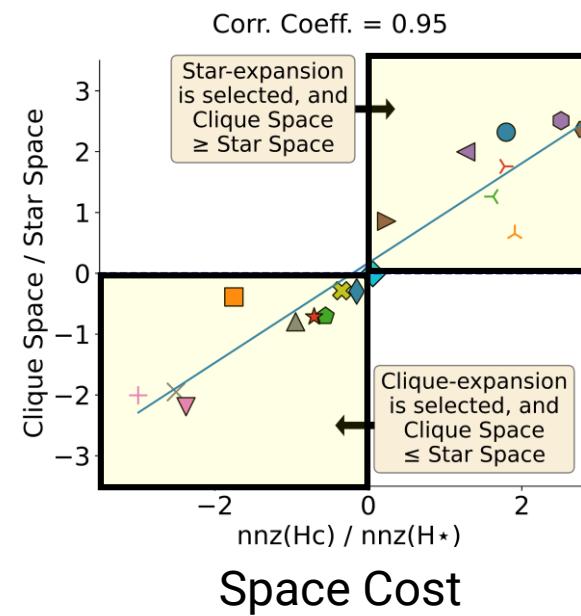
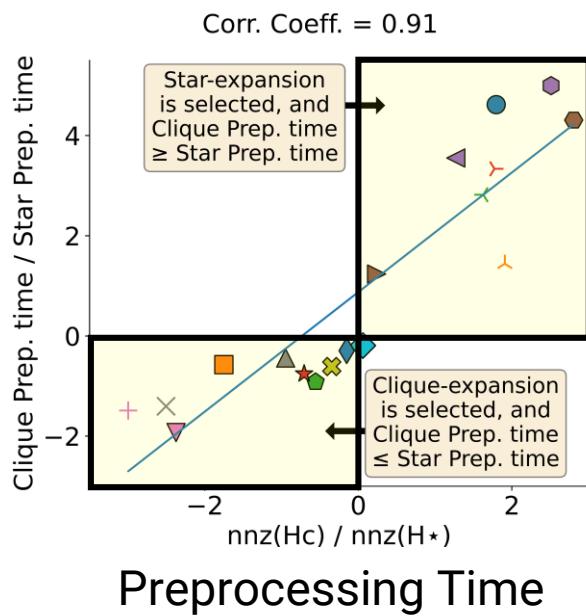


Q4. Automatic Selection Method

- Q4. How precisely does our automatic selection strategy decide more efficient computation method for a given hypergraph?

Datasets

● AM	✖ ML10	● TW	✚ EEU	✖ COD	◆ COH	◆ THM	● WAL	▼ SB
✚ ML1	✖ ML20	◆ YP	✖ EEN	◆ COG	■ THS	★ THU	● TRI	▲ HB



■ : ARCHER selected the efficient method

Q4. Automatic Selection Method (cont.)

- Q4. How precisely does our automatic selection strategy decide more efficient computation method for a given hypergraph?
- We compare proposed non-zero ratio with 3 baselines

Measure	Accuracy (18 datasets)
Non-zero ratio (proposed)	0.944
Density: $ E / V $	0.722
Overlapness: $\sum_{e \in E} e / V $	0.667
Average hyperedge size: $\sum_{e \in E} e / E $	0.889

Roadmap

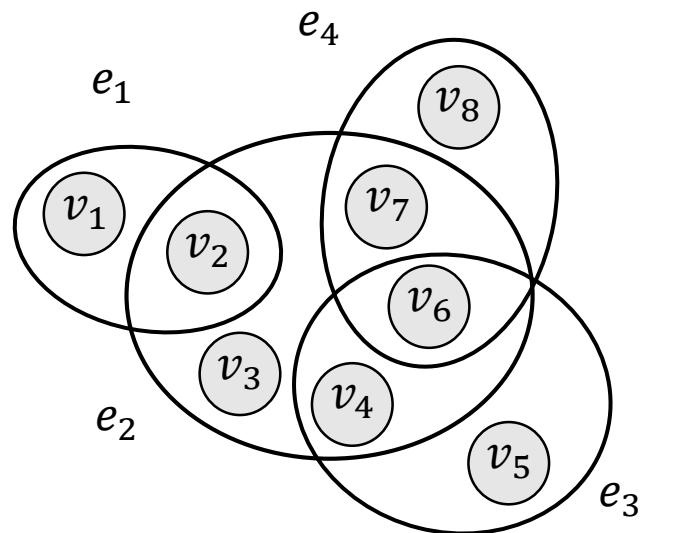
- Overview
- Random Walk with Restart (RWR) on Hypergraphs
- Proposed Algorithm: ARCHER
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- Application: Anomaly Detection <<
- Conclusion



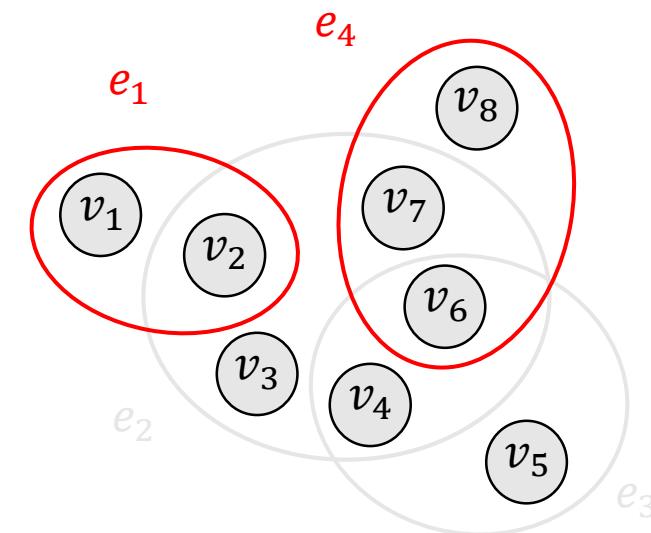
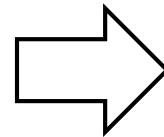
Application: Anomaly Detection

Task Description

- Given a hypergraph, detect **anomalous** hyperedges that deviate from ordinary hyperedges



Hypergraph



Anomaly Detection

Application: Anomaly Detection (cont.)

- We expanded anomaly detection on bipartite graphs (ICDM'05) to hypergraphs
- Intuition: if a hyperedge e is **normal**, **proximity between the nodes** in e should be high
- Normality score

$$ns(e) = \frac{1}{|e|(|e| - 1)} \sum_{u \in e} \sum_{v \in e \setminus \{u\}} r_{u \rightarrow v}$$

$r_{u \rightarrow v}$: RWR score of node v w.r.t. the query node u

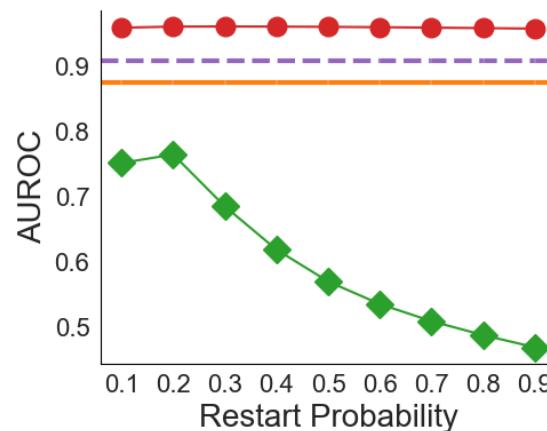
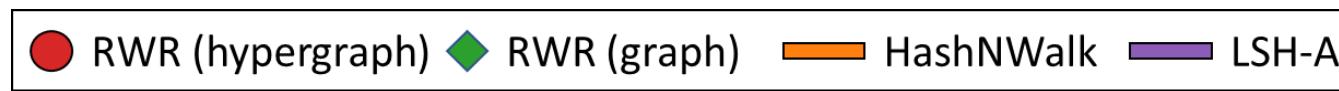
- **ARCHER** accelerates the computation of the normality score

Application: Anomaly Detection (cont.)

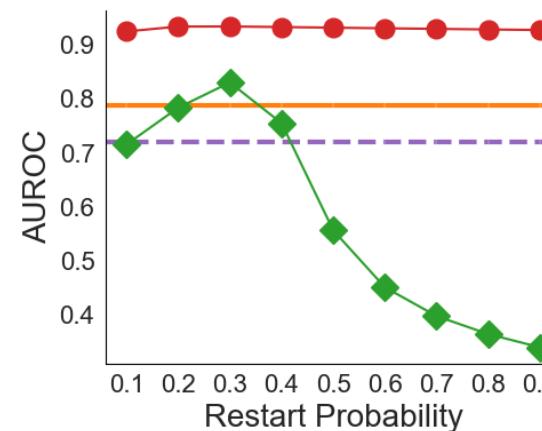
- Experimental Settings
- Datasets: EEN (email-Enron), SB (senate-bills), HB (house-bills)
- Anomalous hyperedge injection: unexpected hyperedges (IJCAI'22)
- Methods
 - RWR on hypergraphs
 - RWR on (pair-wise) graphs
 - LSH-A (COMPLEX NETWORKS'17)
 - HashNWalk (IJCAI'22)

Application: Anomaly Detection (cont.)

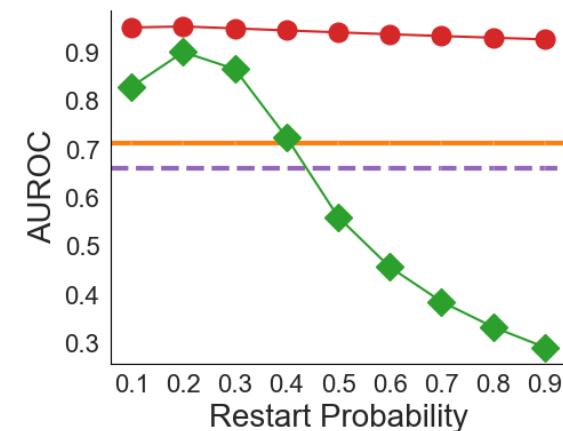
- Q5. Can we achieve more accurate anomaly detection on hypergraphs using RWR scores, compared to existing approaches?



AUROC in EEN



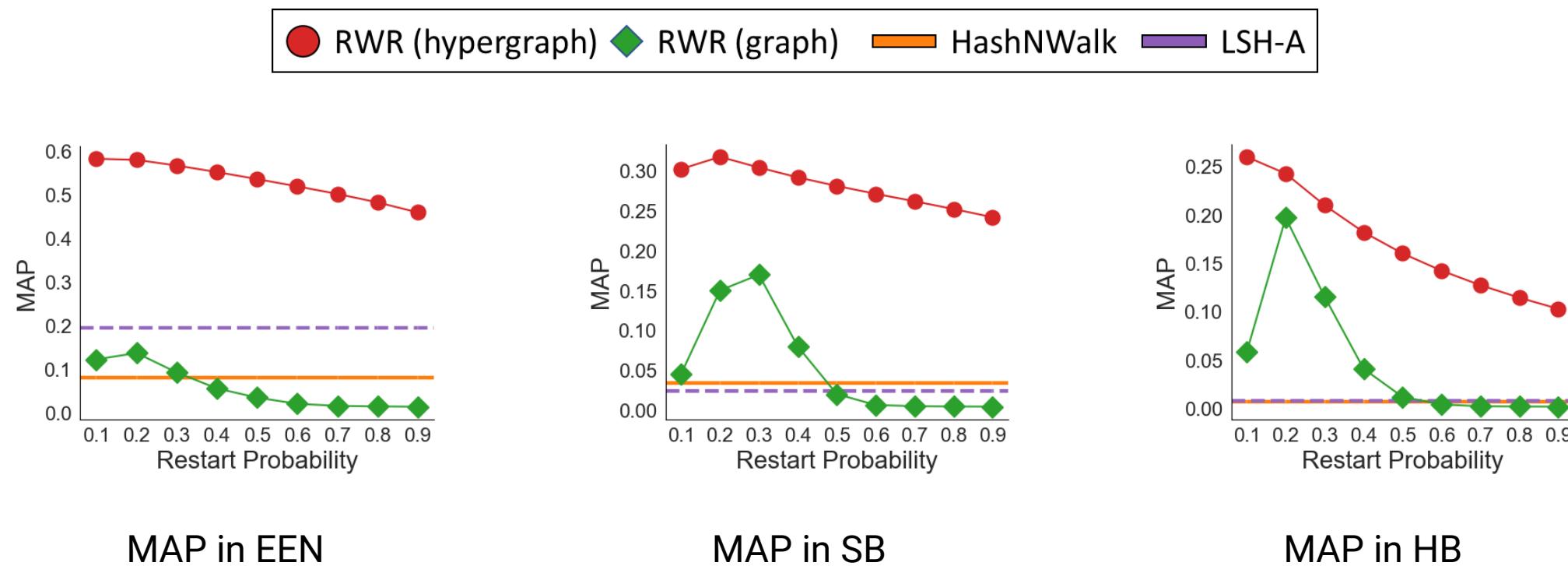
AUROC in SB



AUROC in HB

Application: Anomaly Detection (cont.)

- Q5. Can we achieve more accurate anomaly detection on hypergraphs using RWR scores, compared to existing approaches?

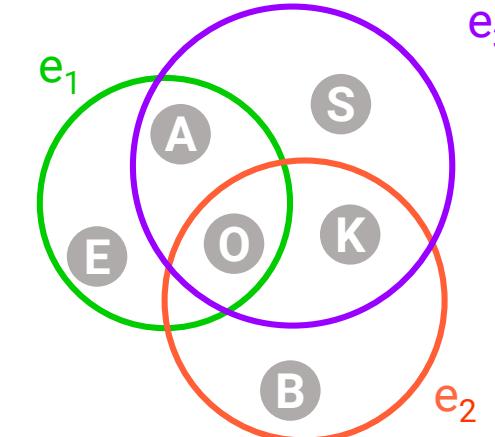
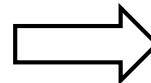
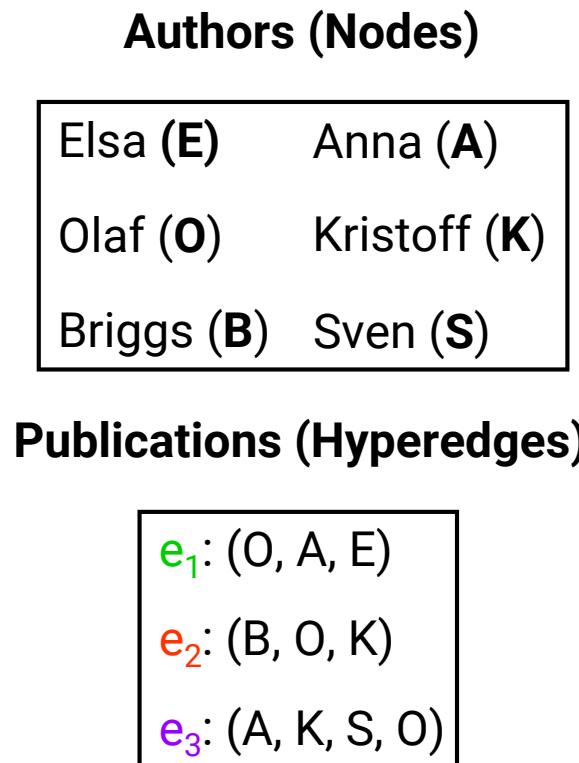


Additional Contents

- There are some additional contents in the paper including:
 - **Edge-dependent node weight (EDNW)**
 - Another application: node retrieval

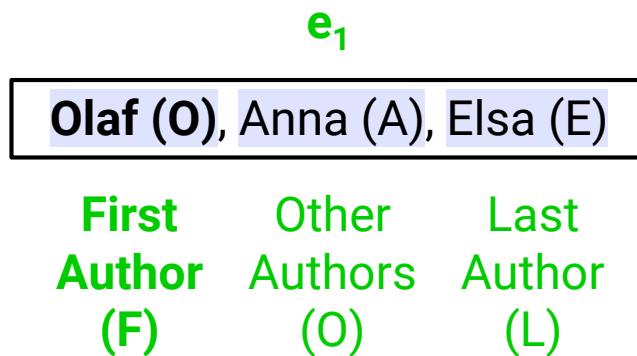
Additional Contents: EDNW

- Let's suppose a co-authorship hypergraph

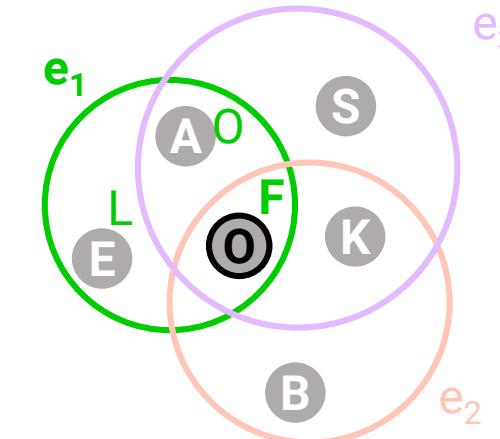


Additional Contents: EDNW (cont.)

- Let's suppose a co-authorship hypergraph

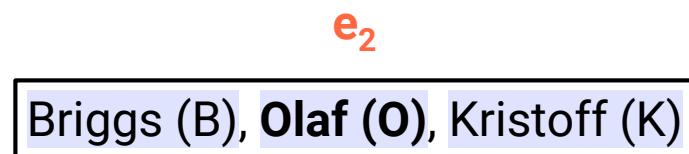


Contribution of a node may vary depending on the hyperedge

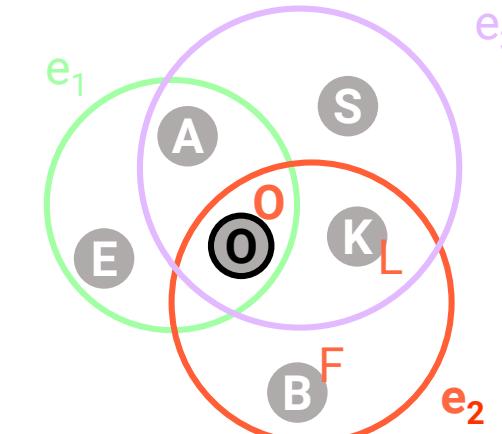


Additional Contents: EDNW (cont.)

- Let's suppose a co-authorship hypergraph

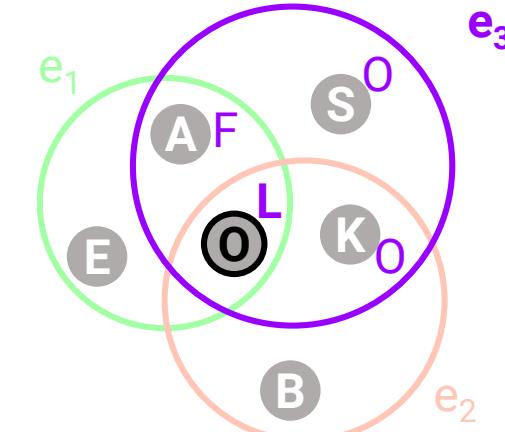
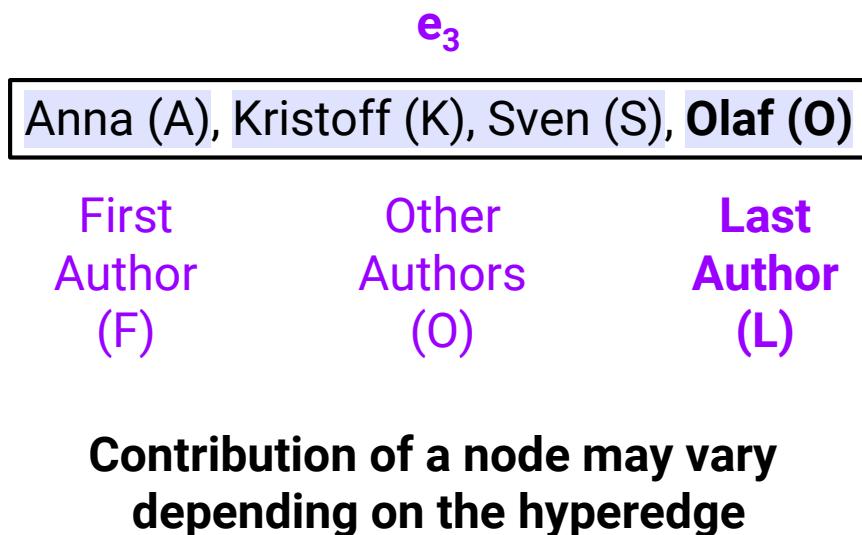


Contribution of a node may vary depending on the hyperedge



Additional Contents: EDNW (cont.)

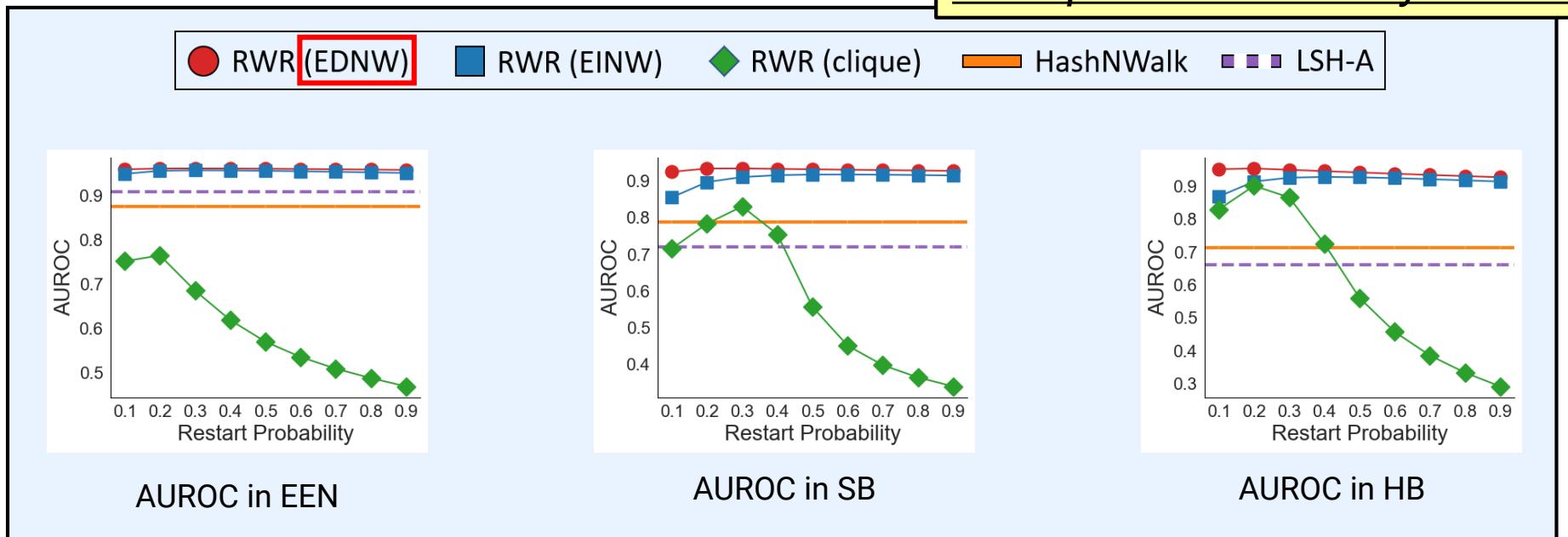
- Let's suppose a co-authorship hypergraph



Additional Contents: EDNW (cont.)

- **ARCHER** can compute RWR on hypergraphs with **EDNW**
- We also show usefulness of **EDNW** in **applications**
 - Anomaly detection
 - Node retrieval

Example with Anomaly Detection



Roadmap

- Overview
- Random Walk with Restart (RWR) on Hypergraphs
- Proposed Algorithm: ARCHER
- Empirical Evaluation of ARCHER
- Application: Anomaly Detection <<
- Conclusion <<



Conclusions

- Our contributions are summarized as follows:

- ✓ We proposed **two RWR computation methods** on hypergraphs
- ✓ We proposed **ARCHER**, which adaptively selects between the two computation methods efficiently
- ✓ We proposed **an application to anomaly detection** using RWR



Paper: <https://doi.org/10.1007/s10618-023-00995-9>



Github repo: <https://github.com/jaewan01/ARCHER>

References

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- **[SIGMOD'15]:** Shin, Kijung, et al. "Bear: Block elimination approach for random walk with restart on large graphs." Proceedings of the 2015 ACM SIGMOD international conference on management of data. 2015.
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- **[ICESS'19]:** Zhang, Ying, Zhiqiang Zhao, and Zhuo Feng. "Towards scalable spectral sparsification of directed graphs." 2019 IEEE International Conference on Embedded Software and Systems. IEEE, 2019.

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- **[CIKM'20]**: Hayashi, Koby, et al. "Hypergraph Random Walks, Laplacians, and Clustering." Proceedings of the 29th ACM international conference on information & knowledge management. 2020.

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- **[IJCAI'22]**: Lee, Geon, Minyoung Choe, and Kijung Shin. "HashNWalk: Hash and Random Walk Based Anomaly Detection in Hyperedge Streams." 31st International Joint Conference on Artificial Intelligence. 2022.
- **[COMPLEX NETWORKS'17]**: Ranshous, Stephen, Mandar Chaudhary, and Nagiza F. Samatova. "Efficient outlier detection in hyperedge streams using minhash and locality-sensitive hashing." *Complex Networks & Their Applications VI: Proceedings of Complex Networks 2017*. Springer International Publishing, 2018.