A Survey on Centrality and Importance Measures in Hypergraphs: Categorization and Empirical Insights

Jaewan Chun[*1], **Fanchen Bu**[*2], **Yeongho Kim**[1], **Atsushi Miyauchi**[3], **Francesco Bonchi**[3], **and Kijung Shin**[†1]

[1]Kim Jaechul Graduate School of AI, KAIST, Seoul, South Korea
[2]School of Electrical Engineering, KAIST, Daejeon, South Korea
[3]CENTAI Institute, Turin, Italy

Abstract

Identifying central entities and interactions is a fundamental problem in network science. While well-studied for graphs (pairwise relations), many biological and social systems exhibit higher-order interactions best modeled by hypergraphs. This has led to a proliferation of specialized hypergraph centrality measures, but the field remains fragmented and lacks a unifying framework.

This paper addresses this gap by providing the first systematic survey of 39 distinct measures. We introduce a novel taxonomy classifying them as: (1) structural (topology-based), (2) functional (impact on system dynamics), or (3) contextual (incorporating external features). We also present an experimental assessment comparing their empirical similarity and computation time. Finally, we discuss applications, establishing a coherent roadmap for future research in this area.

Keywords: Higher-order Networks, Hypergraphs, Centrality Measures, Importance Measures

1 Introduction

Identifying key entities or interactions is a foundational task in network science [114]. Historically, this problem has been studied through the lens of *centrality* [47], a concept rooted in network science to quantify the *structural prominence* of a node or edge based on its position in a graph. Classical measures — such as degree [149], closeness [16], and betweenness [59] centrality — are cornerstones of this tradition. Over time, researchers have also sought to move beyond purely structural prominence, developing measures that capture the *importance* [109] of each entity in shaping system-level behaviors such as network robustness [61], information diffusion [123], or collective dynamics [171].

[*]Both authors contributed equally to this survey.
[†]Corresponding author: `kijungs@kaist.ac.kr`

Centrality vs. Importance

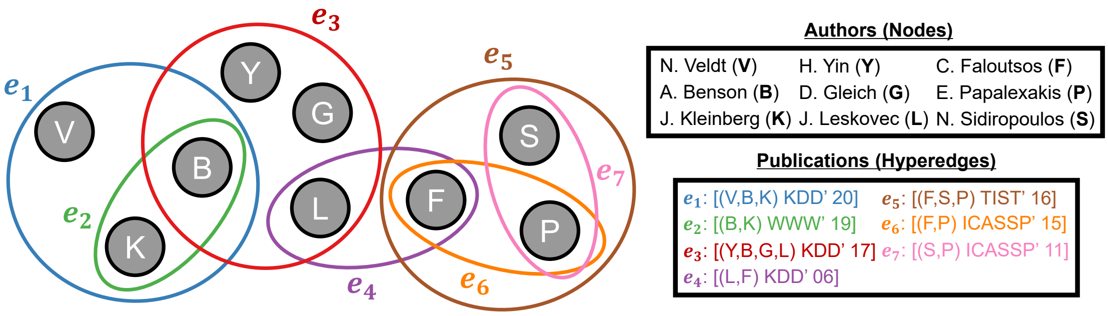
Originated in network science, the notion of *centrality* was traditionally quantifying the structural prominence of nodes or edges within a graph. In parallel, the notion of *importance* has often been used more broadly to capture functional criticality. While there is a part of the literature that use the two terms interchangeably as they were synonyms, recognizing the evolution over time of the term "centrality" beyond its original structural connotation, others consider centrality as a structure-only type of importance, thus with centrality being a narrower and specialized notion of importance. Acknowledging the existence of this ambiguity, in this survey, we do not take a stance in the debate or aim to fix the ambiguity; instead we fully embrace it: We adopt an inclusive perspective and comprehensively review all measures that assign scores of centrality or importance in hypergraphs, without attempting at drawing the borders dividing the two notions.

While these concepts were originally formulated for *pairwise graphs*, where interactions occur between exactly two entities, many real-world systems—from scientific collaborations and communication platforms to biochemical and ecological processes—exhibit *higher-order interactions* that involve groups of entities simultaneously [18, 22, 23]. Overlooking these higher-order interactions can lead to incomplete or misleading conclusions, as it overlooks the shared context and simultaneous nature of group activities. For example, modeling a group email as separate messages misses the shared context of a single communication; reducing a multi-author paper to a set of two-person partnerships hides the team's synergy; and analyzing a complex biochemical process in pairs overlooks the key catalytic event that involves several molecules at the same time.

The shift from dyadic to higher-order structures requires extending classical notions of centrality and importance to *hypergraphs*, which naturally represent multiway relations by allowing hyperedges to connect an arbitrary number of nodes [15]. This transition raises unique challenges, e.g., multiple non-equivalent ways to define paths and distances [164], and ambiguity in how influence or connectivity propagates across multiway links [38]. On one hand, these complexities hinder straightforward generalizations of graph-based measures. On the other hand, they also open opportunities for designing novel frameworks tailored to higher-order systems.

Fig. 1 offers a concrete example of a real-world hypergraph, illustrating how higher-order modeling naturally arises in practical systems, e.g., co-authorship networks here. In this representation, each hyperedge corresponds to a publication and can include any number of nodes (i.e., authors), demonstrating the size flexibility that distinguishes hypergraphs from pairwise graphs where interactions are strictly dyadic. This flexibility enables the model to preserve group-level structure that would otherwise be lost under pairwise reductions. Fig. 1 also previews a key theme of this survey: different centrality and importance measures often emphasize distinct characteristics and perspectives. As shown in the centrality table (Table 1(b)), different measures, such as degree, closeness, and eigenvector centralities, may highlight different nodes as most central. This diversity motivates our systematic examination of the many measures developed for hypergraphs and the distinct analytical perspectives they encode.

As foreshadowed in Fig. 1, researchers have proposed a wide variety of centrality and importance measures tailored specifically for hypergraphs. These range from extensions of classical measures—such as degree, closeness, and betweenness centralities—to fundamentally new frameworks leveraging spectral methods [150], perturbation analyses [75], and cooperative game theory [152]. Despite rapid progress, the literature remains fragmented, lacking a unified framework that systematically categorizes measures based on underlying principles. Such unification would reveal common high-level goals and assumptions of the measures, facilitate meaningful empirical comparisons, and guide future researchers in selecting or developing measures based on such goals and assumptions.



(a) Example hypergraph constructed from a co-authorship network. Nodes represent authors and hyperedges represent publications containing the corresponding sets of authors.

Measure \ Node	**V**	**B**	**K**	**Y**	**G**	**L**	**F**	**P**	**S**
Degree (M1)	0.056	**0.167**	0.111	0.056	0.056	0.111	**0.167**	**0.167**	0.111
Neighbor-degree (M2)	0.077	**0.192**	0.077	0.115	0.115	0.154	0.115	0.077	0.077
Closeness (M5)	0.088	0.131	0.090	0.112	0.112	**0.149**	0.131	0.095	0.093
Betweenness (M6)	0.000	0.269	0.006	0.000	0.000	**0.335**	0.304	0.060	0.025
Eigenvector (M10)	0.029	0.100	0.053	0.046	0.046	0.098	0.226	**0.238**	0.164
Hypercoreness (M18)	0.083	0.083	0.083	0.083	0.083	0.083	**0.167**	**0.167**	**0.167**

(b) Node centrality values in the example hypergraph, with the highest-scoring node(s) under each measure shown in **bold**.

Figure 1: (a) Example co-authorship hypergraph and (b) corresponding node centrality and importance values. Different measures emphasize different structural roles, resulting in distinct rankings. For example, betweenness centrality (M6) highlights author L due to its bridging position across multiple publication groups, whereas hypercoreness (M18) assigns the highest scores to F, P, and S, reflecting the tightly connected triadic region they form.

This survey aims to fill that gap by providing the first dedicated and systematic overview of centrality and importance measures in hypergraphs. We introduce a taxonomy that distinguishes three broad categories—*structural*, *functional*, and *contextual* measures—each reflecting a different perspective on what makes an entity "central" or "important." By analyzing underlying intuitions and mathematical foundations, we aim to establish a coherent framework for understanding existing measures and to highlight opportunities for future research. Additionally, we conduct empirical analyses across diverse real-world datasets to provide practical insights.

1.1 Scope and Contributions

Our contributions can be summarized as follows:

- (Sections 3 to 5) We propose a systematic taxonomy of hypergraph centrality and importance measures that organizes 39 measures into three principled categories: (1) **structural measures** that capture centrality based purely on topology; (2) **functional measures** that evaluate importance via system-level behavior such as perturbation or coalition value; and (3) **contextual measures** that incorporate external features, labels, or learned representations.
- (Section 6) We conduct a comprehensive empirical study across diverse real-world hypergraphs to compare representative measures. Our analyses examine (1) their **empirical similarity**, identifying when different methodological choices lead to similar or divergent

rankings, and (2) their **computational behavior**, including run-time complexity and the practicality of heavier path-based or spectral variants. These results provide guidance on selecting measures under accuracy-efficiency tradeoffs.

- (Section 7) We examine **key application domains** where hypergraph centrality and importance play a critical role, including computational biology, social and communication systems, and infrastructure networks. Across these domains, we highlight how different measures capture domain-specific higher-order structures, linking methodological choices to practical modeling needs.

Together, these contributions provide the first comprehensive map of the field, enabling researchers to position existing measures within a unified framework and to design new measures aligned with their analytical goals.

1.2 Related Surveys

The literature on identifying important entities in pairwise graphs is extensive, with numerous surveys covering two related themes. On one hand, many reviews have cataloged the vast family of centrality measures, from classical formulations to modern machine learning-based approaches [24, 47, 64, 145, 165]. On the other hand, the closely related problem of identifying key, critical, or influential nodes has also been thoroughly surveyed, often with a focus on specific domains such as information diffusion in social networks [1, 21, 67, 144], network robustness [35, 184], or biological systems [167].

Simultaneously, while the study of higher-order models has spurred a significant number of surveys on hypergraphs, these works have predominantly focused on topics other than centrality. A large body of literature is dedicated to hypergraph machine learning, with dedicated reviews on representation learning [7, 63], hypergraph neural networks [84, 166, 179], and applications in recommender systems [107]. Other topics that have received comprehensive treatment include hypergraph patterns and generators [100], partitioning algorithms [30], visualization techniques [56], and hyperedge prediction [34].

Despite this wealth of research in parallel fields, a systematic and dedicated review of centrality and importance measures in hypergraphs has been notably absent. The most relevant works are broad surveys on critical node identification in graphs that only briefly touch upon higher-order networks (e.g., [35]). This survey aims to fill this critical gap by providing the first unified and systematic roadmap of this emerging field.

1.3 Paper Outline

The remainder of this paper is structured as follows. Section 2 introduces preliminaries on graphs and hypergraphs, as well as classical centrality measures on pairwise graphs. Section 3, Section 4, and Section 5 contains the survey of respectively structural, functional, and contextual measures of centrality and importance in hypergraphs. Section 6 reports empirical comparisons and discusses insights on representative centrality and importance measures. Section 7 surveys real-world applications of hypergraph centrality and important measures across diverse domains, including social, biological, communication, and infrastructure systems. Section 8 contains our concluding remarks outlining the road ahead and future research opportunities. Fig. 2 presents the taxonomy of measures covered in this survey, which is also a navigable table of contents for Section 3, 4, and 5.

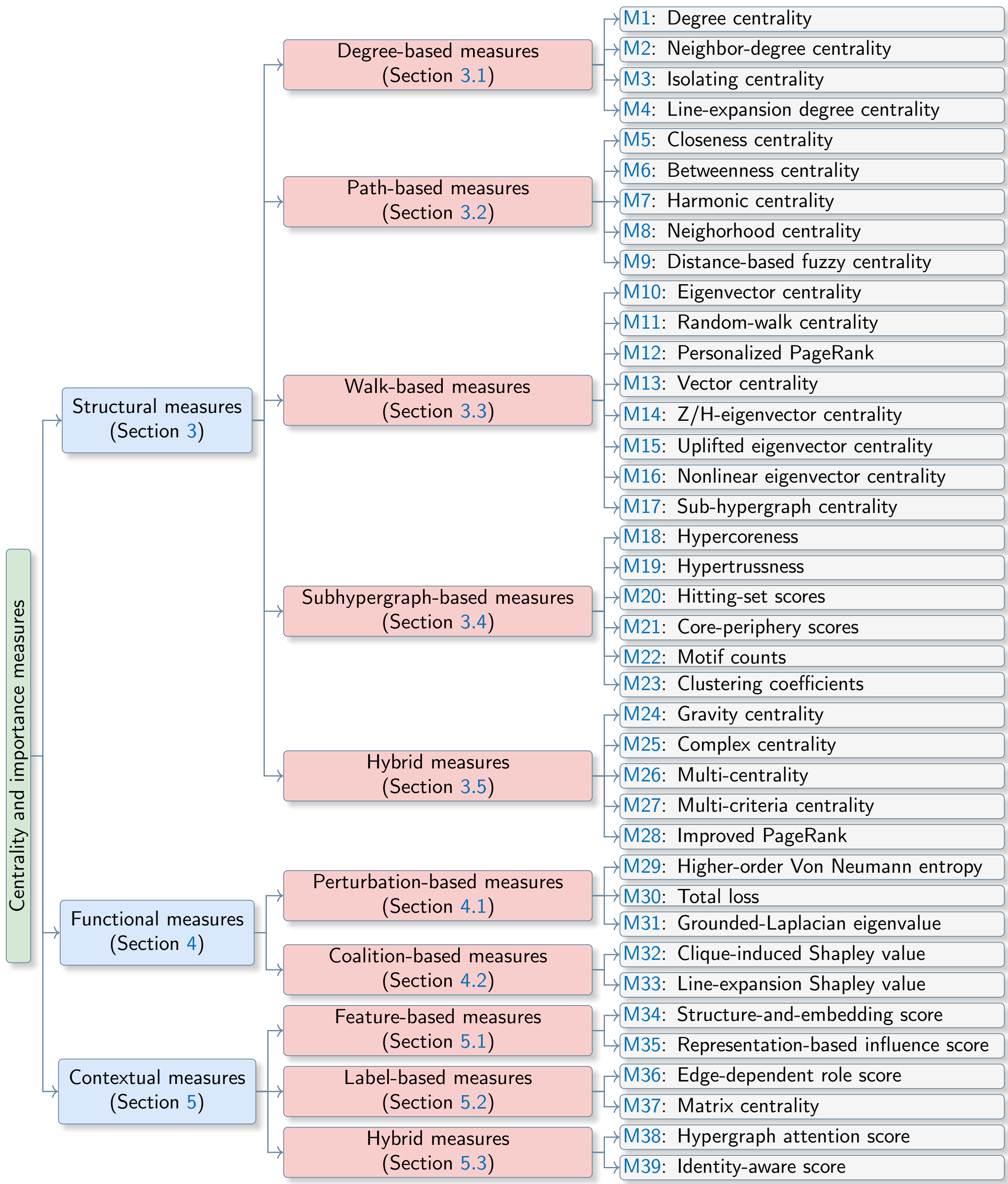


Figure 2: **Taxonomy of hypergraph centrality and importance measures.** The framework distinguishes three major categories: structural measures, which rely on the combinatorial structure of the hypergraph; functional measures, which evaluate importance in terms of system behavior; and contextual measures, which integrate information beyond structure. Each category is further divided into sub-categories that capture specific methodological choices.

2 Preliminaries

In this section, we present the preliminaries and notations used throughout the paper. We first define hypergraphs and their expansions to pairwise graphs, and then briefly review centrality measures in pairwise graphs as a foundation for their generalizations to hypergraphs.

2.1 Hypergraphs

First, we introduce the concept of hypergraphs, together with some related concepts.

P1. Hypergraphs. A *hypergraph* is defined as $H = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_n\}$ is the set of nodes and $\mathcal{E} = \{e_1, \ldots, e_m\}$ is the set of hyperedges, with $n = |\mathcal{V}|$ being the number of nodes and $m = |\mathcal{E}|$ the number of hyperedges. Each hyperedge $e \in \mathcal{E}$ is a non-empty subset of $\mathcal{V}$, i.e., $e \subseteq \mathcal{V}$ and $e \neq \emptyset$. Refer to Fig. 1a for an example.

P2. Incident hyperedges and constituent nodes. For a node v and a hyperedge e, if $v \in e$, we call e an *incident hyperedge* of v, and v a *constituent node* of e.

P3. Node degrees. The *degree* of a node v, $\deg(v) := |\{e \in \mathcal{E} \mid v \in e\}|$, is the number of its incident hyperedges.

P4. Neighbors and neighbor-degrees. The set of *neighbors* of a node v, $\mathcal{N}(v) := \{v \neq u \in \mathcal{V} \mid \exists e \in \mathcal{E}, \{u, v\} \subseteq e\}$, consists of nodes that co-occur in at least one hyperedge with v. The number of neighbors, $|\mathcal{N}(v)|$, is called the *neighbor-degree* of v.

P5. Hyperedge sizes. The *size* of a hyperedge e, $s_e := |e|$, is the number of its constituent nodes, which is also sometimes referred to as the *degree* of the hyperedge. In this work, we consider hypergraphs with each hyperedge of size at least two, i.e., $|e| \geq 2, \forall e \in \mathcal{E}$.

P6. Subhypergraphs. A hypergraph $H' = (\mathcal{V}', \mathcal{E}')$ is a *subhypergraph* of another hypergraph $H = (\mathcal{V}, \mathcal{E})$ if the node and hyperedge sets of H' are subsets of those of H, respectively, i.e., $\mathcal{V}' \subseteq \mathcal{V}$ and $\mathcal{E}' \subseteq \mathcal{E}$.

P7. Incidence matrix. The *incidence matrix* of a hypergraph H, $\mathbf{H} = \mathbf{H}(H) \in \mathbb{R}^{n \times m}$, summarizes the membership information. For the commonly considered unweighted case, the entries are binary, where $\mathbf{H}_{ij} = 1$ if node v_i is in hyperedge e_j, i.e., $v_i \in e_j$, and $\mathbf{H}_{ij} = 0$ otherwise. This can be extended to weighted hypergraphs, where the entries of $\mathbf{H}$ can take non-binary values to represent the strengths or weights of membership.

P8. Uniform hypergraphs. A hypergraph $H = (\mathcal{V}, \mathcal{E})$ is *r-uniform* if all hyperedges are of size r, i.e., $|e| = r, \forall e \in \mathcal{E}$.

P9. Pairwise graphs. A 2-uniform hypergraph is specifically called a *pairwise graph*, where each edge consists of exactly two nodes. When the context is clear, we may simply use *graphs* to refer to pairwise graphs.

2.2 Connectivity-Related Concepts

We now introduce concepts related to connectivity, e.g., walks, paths, and distances.

P10. Adjacent hyperedges. Given a pairwise graph $G = (\mathcal{V}, \mathcal{E})$, two different edges $e \neq e' \in \mathcal{E}$ are *adjacent* if $e \cap e' \neq \emptyset$. One can straightforwardly extend this concept to a hypergraph $H = (\mathcal{V}, \mathcal{E})$, where two different hyperedges $e \neq e' \in \mathcal{E}$ are *adjacent* if $e \cap e' \neq \emptyset$. Recall that each edge in a pairwise graph contains exactly two nodes (see P9), so if two (non-identical) edges are adjacent, they must have exactly one node in common. By contrast, in hypergraphs, a hyperedge can contain an arbitrary number of nodes, which implies that two hyperedges may intersect at multiple nodes, which allows more flexibility in the definition of *adjacent hyperedges*. Indeed, researchers [2, 14, 113, 158] have considered the definition of *s-adjacent* hyperedges: Two hyperedges e and e' are s-adjacent for some $s \in \mathbb{N}$ if they have at least s nodes in common, i.e., $|e \cap e'| \geq s$. When $s = 1$, this definition reduces to the aforementioned straightforward generalization. Researchers [94, 112, 189] have also considered using a proportional threshold (e.g., two hyperedges e and e' are considered adjacent if $|e \cap e'| / \min(|e|, |e'|) \geq p$ for some proportional threshold $p \in [0, 1]$). Rather than using the size of intersection solely as a threshold, some researchers [28, 29, 98, 125, 164] have also incorporated it as additional information, e.g., as the weight of the adjacency between hyperedges.

On adjacent hyperedges

Existing connectivity-related concepts and centrality measures use different definitions of *adjacent hyperedges*. These concepts and measures usually remain well-defined and meaningful regardless of the specific adjacency definition. Therefore, in subsequent discussions of related concepts and centrality measures, we will use the term "adjacent hyperedges" without explicitly specifying the adjacency criterion, unless otherwise noted.

P11. Walks. Given a hypergraph $H = (\mathcal{V}, \mathcal{E})$, a *walk* of length k is a sequence of nodes $(v_0, v_1, \ldots, v_k)$ such that there exists a sequence of hyperedges $(e_1, e_2, \ldots, e_k)$ satisfying: (1) each pair of consecutive nodes v_{i-1} and v_i coexists in the hyperedge e_i, i.e., $\{v_{i-1}, v_i\} \subseteq e_i$ for each $i \in \{1, \ldots, k\}$, and (2) every pair of consecutive hyperedges is adjacent, i.e., e_{i-1} and e_i are adjacent, for each $i \in \{2, \ldots, k\}$. In a walk, the same node can appear multiple times, i.e., it is allowed that $v_i = v_j$ for some $i \neq j$.

P12. Paths. A *path* of length k is a walk $(v_0, v_1, \ldots, v_k)$ where all nodes are distinct, i.e., $v_i \neq v_j, \forall i \neq j$.

P13. Shortest paths and distances. For two nodes $u, v \in \mathcal{V}$, a *shortest path* between them is a path $(v_0, v_1, \ldots, v_k)$ of minimum length k among all paths connecting u and v, i.e., where $v_0 = u$ and $v_k = v$. The minimum length k is called the *distance* $\delta(u, v)$ between u and v. There are possibly multiple shortest paths of the same length between the same node pair. If no path exists between u and v, we set $\delta(u, v) = \infty$.

2.3 Expansions from Hypergraphs to Pairwise Graphs

Researchers have considered several *expansions* of hypergraphs, i.e., projections of hypergraphs into pairwise graphs, to borrow results from pairwise-graph analysis for hypergraph analysis.

P14. Clique expansion. Given a hypergraph $H = (\mathcal{V}, \mathcal{E})$, its *clique expansion* [155] $G_C = (\mathcal{V}_C, \mathcal{E}_C)$ is generated by replacing each hyperedge by a clique over its constituent nodes, i.e., $\mathcal{V}_C = \mathcal{V}, \mathcal{E}_C = \{\{u, v\} \mid u, v \in e, e \in \mathcal{E}\}$. This reduces H to a weighted or unweighted pairwise graph, depending on whether edge multiplicities are considered.

P15. Star expansion. Given a hypergraph $H = (\mathcal{V}, \mathcal{E})$, its *star expansion* [190] $G_S = (\mathcal{V}_S, \mathcal{E}_S)$ is generated by combining sets of nodes and hyperedges as a set of nodes (i.e., $\mathcal{V}_S = \mathcal{V} \cup \mathcal{E}$) and putting an edge between a hyperedge and each of its constituent nodes (i.e., $\mathcal{E}_S = \{\{v, e\} \mid v \in e, v \in \mathcal{V}, e \in \mathcal{E}\}$).

P16. Line expansion. Given a hypergraph $H = (\mathcal{V}, \mathcal{E})$, its *line expansion* [20, 173] is a pairwise graph $G_L = (\mathcal{V}_L, \mathcal{E}_L)$ generated by treating original hyperedges as nodes, i.e., $\mathcal{V}_L = \mathcal{E}$ and putting an edge between two nodes (corresponding two hyperedges) if and only if they are adjacent in the original hypergraph. As mentioned above in Section 2.2, there exist various definitions of "adjacent hyperedges", and they give different definitions of line expansion.

2.4 Basic Centrality and Importance Measures in Pairwise Graphs

Given a pairwise graph $G = (\mathcal{V}, \mathcal{E})$, a centrality or importance measure for nodes (resp. edges) is a function that assigns a score to each node (resp. edge), aiming to quantify its importance in the network. Following Boldi and Vigna [25], we briefly review several classical graph centrality measures that serve as the conceptual basis for hypergraph generalizations. Although this survey broadly covers both centrality and importance measures, here, we focus our review on a curated set of foundational centrality measures from the pairwise graph literature. These classical measures—such as degree, closeness, and betweenness—not only form the historical bedrock of the field but also provide the essential conceptual vocabulary for the hypergraph measures that follow. Many of the higher-order measures to be discussed later are direct generalizations or sophisticated adaptations of these original ideas. We will adopt a more inclusive perspective in the main body of the survey, covering a broader spectrum of functional importance measures and exploring the complexities of hypergraphs. See also, e.g., [21, 60, 89, 144, 145], for comprehensive surveys on centrality and importance measures in pairwise graphs.

P17. Degree centrality. *Degree centrality* [149] measures the immediate connectedness of a node: $C_{\text{deg}}(v) = \deg(v)$, where $\deg(v)$ is the number of edges incident to v (see P3). Sometimes, this measure is normalized by the number of nodes in the entire graph, i.e., $\tilde{C}_{\text{deg}}(v) = \deg(v)/|\mathcal{V}|$, which does not affect the relative relations in the same graph.

P18. Closeness centrality. *Closeness centrality* [16] measures how close a node is to the other nodes, specifically, the inverse of the sum of distances (see P12) to all other nodes: $C_{\text{cls}}(v) = \left(\sum_{u \in \mathcal{V} \setminus \{v\}} \delta(v,u)\right)^{-1}$. A limitation of this measure is that the scores of all nodes become zero if the graph is disconnected (i.e., if not all node pairs have finite distances).

P19. Harmonic centrality. *Harmonic centrality* [25] is similar to closeness centrality, but it measures the sum of the inverses instead of the inverse of the sum, so that it overcomes the above limitation: $C_{\text{hmn}}(v) = \sum_{u \in \mathcal{V} \setminus \{v\}} \delta(v,u)^{-1}$.

P20. Betweenness centrality. *Betweenness centrality* [59] measures how often a node appears on the shortest paths (see P13). Specifically, it is the sum of the fractions of shortest paths that pass through the node, taken over all pairs of other nodes: $C_{\text{btw}}(v) = \sum_{s,t \in \mathcal{V} \setminus \{v\}} \sigma_{st}(v)/\sigma_{st}$, where σ_{st} is the number of all shortest paths between s and t, and $\sigma_{st}(v)$ is the number of those passing through v.

P21. Eigenvector centrality. *Eigenvector centrality* [146] is a spectral measure of a node's influence within a network. It is calculated using the graph's adjacency matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$, where the entry $\mathbf{A}_{ij}$ represents the weight of the edge between nodes v_i and v_j (for unweighted cases, $\mathbf{A}_{ij} = 1$ if an edge exists and 0 otherwise). The centrality score of each node v_i is given by the i-th component of the *principal eigenvector* $\boldsymbol{c}$ of $\mathbf{A}$, which is the eigenvector corresponding to the largest eigenvalue. This formulation defines a node's score as being proportional to the sum of the scores of its neighbors, i.e., a node is considered more important if it is connected to other important nodes.

P22. Katz centrality. *Katz centrality* [79] extends the core idea of eigenvector centrality. Instead of only considering a node's immediate neighbors, it defines a node's importance by counting the total number of walks of all lengths that terminate at that node, while attenuating longer paths with exponential decays: $C_{\text{katz}}(v_i) = \sum_{k=1}^{\infty} \sum_{j=1}^{n} \alpha^k (\mathbf{A}^k)_{ji}$, where $\alpha > 0$ is a damping factor. The term $(\mathbf{A}^k)_{ji}$ is the number of walks of length k from node v_j to node v_i, and $\sum_{j=1}^{n} \alpha^k (\mathbf{A}^k)_{ji}$ thus counts the total number of walks of length k ending at node v_i.

P23. PageRank centrality. *PageRank* [126] defines a node's importance as its stationary probability under a "random surfer" model. This surfer either follows a random edge from its current node (with probability α) or "teleports" to a new node in the graph (with probability $1-\alpha$). The vector of PageRank scores, $\boldsymbol{c}$, is the stationary distribution of this process, satisfying the recursive equation: $\boldsymbol{c} = \alpha \mathbf{P}^{\top} \boldsymbol{c} + (1-\alpha)\boldsymbol{v}$, where $\mathbf{P}$ is the row-stochastic transition matrix,[1] and

[1]The entry P_{ij} of the transition matrix $\mathbf{P}$ specifies the probability of the random surfer moving from node i to node j. The matrix is row-stochastic (i.e., each of its rows sums to one) because the sum of probabilities for all possible destinations from each node i must be 1.

v is the teleportation distribution, which is typically uniform. If v is biased toward a specific set of nodes, the measure is referred to as *Personalized PageRank* [74].

Generalizing the above measures to hypergraphs, or designing new ones tailored to hypergraph structures, is often non-trivial due to the inherently non-dyadic nature of hypergraph interactions. These challenges have motivated a rich and diverse landscape of hypergraph centrality measures, which this survey proceeds to categorize and analyze.

2.5 Taxonomy of Hypergraph Centrality and Importance Measures: Overview

In this work, we provide a comprehensive survey of existing hypergraph centrality and importance measures by organizing them into a systematic categorization that clarifies their underlying principles and points of departure. We propose a framework that divides these measures into three major categories, each with several sub-categories: (1) *structural measures* (Section 3), which quantify importance based purely on the *static, combinatorial topology* of the hypergraph (e.g., via degrees, paths, walks, or subhypergraphs); (2) *functional measures* (Section 4), which evaluate importance by an entity's *impact on system-level dynamics or behavior*, often assessed through perturbation analysis (e.g., the effect of removing nodes or hyperedges) or cooperative game theory (e.g., by quantifying the contribution of a node or hyperedge to the value of coalitions it can form with others); and (3) *contextual measures* (Section 5), which incorporate additional information beyond structure, such as features, labels, or hybrid combinations thereof.

In the sections that follow, we examine each of these three categories. Each category is further organized into several sub-categories. For each of these sub-categories, we articulate its defining intuition and review representative measures. This structured overview is designed to situate individual contributions within a unified taxonomy (see Fig. 2) and to illuminate the foundational principles that connect otherwise disparate measures.

3 Structural Measures

Structural measures form the most fundamental class of hypergraph centrality and importance measures, relying solely on the static topology of hypergraphs. They determine importance from *primary structural elements* such as node degrees, hyperedge sizes, paths, walks, or subhypergraphs (see Section 2), and capture how the combinatorial arrangement of higher-order connections shapes the prominence of individual nodes or hyperedges.

3.1 Degree-based Measures

The most straightforward and intuitive structural elements that can be used for calculating measures are *degrees*, including node degrees (see P3) and neighbor-degrees (see P4).[2] Degree-based measures typically assume that nodes/hyperedges that are *connected to more nodes/hyperedges* are more central and important, and thus are assigned higher centrality scores.

M1. Degree centrality. Faust [53] extended degree centrality (see P17) to hypergraphs for both nodes and hyperedges. The *degree centrality* of each node v is its degree $\deg(v)$ (see P3), and that of each hyperedge e is its size $|e|$ (see P5). This measure is equivalent to computing the degrees in the star expansion (see P15). Weights on nodes and/or hyperedges can be naturally included. Specifically, the degree centrality of each node can be the sum of weights of its incident hyperedges (see P2), and that of each hyperedge can be the sum of weights of its constituent nodes (see P2). For example, Kapoor et al. [77] considered degree centrality for

[2] Note that node degrees and neighbor-degrees are equivalent for pairwise graphs (more precisely, they are equivalent for pairwise graphs that are undirected and unweighted, and without self-loops), but they are different in general for hypergraphs.

nodes while allowing different weights on hyperedges.

M2. Neighbor-degree centrality. Similarly, we can define the *neighbor-degree centrality* for each node as its neighbor-degree $|\mathcal{N}(v)|$ (see P4). This measure is equivalent to the degrees in the unweighted clique expansion (see P14). We can incorporate weights on hyperedges into the definition by considering weighted clique expansion (see, e.g., [49, 84]).

M3. Isolating centrality. Tejaswi et al. [157] proposed *isolating centrality* to measure the importance of nodes. Let $\mathcal{V}_{\min}$ denote a set of nodes with the lowest degree in the hypergraph (how many nodes to choose to put in $\mathcal{V}_{\min}$ is a hyperparameter), the isolating centrality of each node v is the product of (1) the number of its neighbors in $\mathcal{V}_{\min}$ and (2) the total number of its neighbors. The intuition is that a node is considered important if it is not only connected to many nodes, but also safeguards many low-degree nodes. This measure can be seen as a variant of neighbor-degree centrality (see M2) where we assign higher weights to the lowest-degree nodes.

M4. Line-expansion degree centrality. After obtaining the line expansion (see P16), the degrees of nodes in the line expansion (which correspond to hyperedges in the original hypergraph) give a centrality measure for hyperedges. Specifically, the *line-expansion degree centrality* of each hyperedge e is the number of other hyperedges e' that are adjacent (see P10) to e. As discussed in Section 2.2, we can have different variants of this measure by considering different definitions of adjacent hyperedges.

3.2 Path-based Measures

Another structural element commonly used to define centrality and importance measures is *paths* (see P12). The intuition behind these measures typically evaluates either the quality or the quantity of paths associated with an entity. From a quality perspective, nodes/hyperedges are considered more central and important if they lie on *shorter* paths to others, a concept typically captured using distances (see P13). From a quantity perspective, nodes/hyperedges are considered more central and important if they lie on *a greater number* of paths.

Paths and distances in hypergraphs

The key challenge in defining path-based measures on hypergraphs lies in generalizing the notions of paths and distances (see P12). As discussed in Section 2.2, the higher-order nature of hypergraphs allows various definitions of paths and their length. Typically, we consider distances between nodes, and there are two major categories of common approaches in existing literature. The first category uses clique expansion (see P14) or star expansion (see P15) to reduce hypergraphs into pairwise graphs (see Section 2.3), and then considers distances between nodes in the reduced graphs. The second category considers adjacent hyperedges (see P10), obtains distances between hyperedges (which can be interpreted as computing distances between nodes in the line expansion; see P16), and then sets the distance between two nodes as the minimum distance between their respective incident hyperedges, as described in Section 2.2 (see P10 to P13).

M5. Closeness centrality. Faust [53] extended closeness centrality (see P18) for both nodes and hyperedges using star expansion (see P15). Both original nodes and hyperedges become nodes in the star expansion, and the *closeness centrality* of each original node/hyperedge is defined as the closeness centrality of its corresponding node in the star expansion. As mentioned in Section 2.2 and above, we can obtain various definitions of closeness centrality in hypergraphs by considering different definitions of paths and distances. Closeness centrality has also been extended to hypergraphs using clique expansion [164] (see P14) and line expansion [2, 164] (see P16). Similarly to the closeness centrality on graphs (see P18), this measure is meaningful only when the hypergraph is connected, i.e., when all node pairs have finite distances under the corresponding definition of adjacent hyperedges.

M6. Betweenness centrality As another centrality measure based on paths, betweenness centrality (see P20) can also be extended to hypergraphs in various ways by considering different definitions of paths. Researchers have also defined *betweenness centrality* for hypergraphs in various ways using star expansion [53], clique expansion [164], and line expansion [2, 164]. Puzis et al. [133] proposed a variant of betweenness centrality for pairwise graphs with weights on each node pair, which can be naturally extended to hypergraphs. A well-known limitation of betweenness centrality is its high computational complexity since the computation involves calculating the shortest paths between all node pairs. To this end, there are existing techniques for efficient computation or approximation of betweenness centrality in graphs (see, e.g., [10, 11]) and hypergraphs (see, e.g., [134]).

M7. Harmonic centrality. Similarly to closeness centrality (see M5), harmonic centrality (see P19) is also defined by distances, and has been extended to hypergraphs in various ways by considering different definitions of distances. Again, researchers have considered star expansion [50], clique expansion [2], and line expansion [2]. Similar to the harmonic centrality on pairwise graphs (see P19), it has the merit that it is meaningful even on disconnected hypergraphs. Harmonic centrality essentially gives each distance δ a weight $w_\delta = \delta^{-1}$. One can consider more general weights w_δ on different distances. Such ideas have been considered for pairwise graphs (see, e.g., [73] and [46]), and they can be straightforwardly generalized to hypergraphs.

M8. Neighborhood centrality. Amato et al. [4] proposed *neighborhood centrality*, which quantifies the importance of each node as the fraction of nodes in the hypergraph within a distance threshold (which is a hyperparameter). The authors specifically used the distances in line expansion, while in principle, as discussed in Section 2.2 and above, any definition of distances would suffice to give a reasonable definition. This measure can be interpreted as a generalization of neighbor-degree centrality (see M2), extending beyond immediate neighbors to those within a multi-hop radius.

M9. Distance-based fuzzy centrality. Zhang et al. [183] proposed *higher-order distance-based fuzzy centrality*, which quantifies the importance of each node using the entropy on the distribution of distances to other nodes. For each node v, they group the other nodes in the hypergraphs by their distances to v, weight the counts at each distance using distance-dependent fuzzy decays, normalize these weighted counts into a probability distribution, and compute the entropy of that distribution. The high-level idea is that a node is considered more important if it is not only close to many other nodes but also maintains a balanced and diversified influence across multiple distance layers, ensuring both strong local impact and broader spreading potential. The authors specifically used the distances in s-line expansion (see P16), while in principle, as discussed in Section 2.2 and above, any definition of distances would suffice to give a reasonable definition. similar ideas have also been considered by Liu et al. [108].

Why usually only shortest paths?

As we can see above, path-based measures typically rely exclusively on *shortest* paths, rather than general paths. But why are such preferences prevalent? In our understanding, the main reason lies in the computational complexity. Calculating all paths is computationally intensive and often infeasible, as it involves enumerating and analyzing a combinatorially large set of possible paths. On the other hand, shortest paths can be computed efficiently using algorithms such as Dijkstra's, due to their well-defined and limited nature. Specifically, on pairwise graphs, the time complexity of finding all paths between a single node pair is $O(|V|!)$ in the worst case, while that of finding all shortest paths between a node pair is at most $O(|V|^2)$.

3.3 Walk-based Measures

A broader and more expressive class of centrality and importance measures is based on *walks* (see P11). Such measures typically rely on matrix or tensor representations of hypergraphs and extract importance scores as eigenvectors or stationary distributions of associated operators. With such spectral operations, walk-based measures typically assume that a node is important if it is frequently visited by random walks on a hypergraph. This perspective implies that the centrality of a node/hyperedge is reinforced by the importance of the other nodes/hyperedges that constitute the walks leading to it.

M10. Eigenvector centrality. In pairwise graphs, eigenvector centrality (see P21) is one of the most straightforward walk-based measures. Generalizations to hypergraphs have defined centrality for both **nodes** and **hyperedges**. For node centrality, Zhou et al. [186] proposed computing eigenvector centrality on the clique expansion (see P14), while Faust [53] considered the star expansion (see P15). To measure the importance of hyperedges, Kovalenko et al. [91] considered the line expansion (see P16), where the centrality of a node in the expansion corresponds to the centrality of a hyperedge in the original hypergraph.

M11. Random-walk centrality. Chitra and Raphael [38] proposed *random-walk centrality* for hypergraphs, where each node has distinct, edge-dependent weights reflecting its importance within specific hyperedges. This measure is defined via a random walk process: starting from a node, the walker selects the next hyperedge with probability proportional to the hyperedge's weight, and subsequently selects the next node from that hyperedge with probability proportional to its edge-dependent node weight. The final centrality measure of each node corresponds to the stationary distribution of this random walk. Chitra and Raphael [38] also showed that without edge-independent node weights, random walks are equivalent to those on clique expansions. Chun et al. [43] extended this by further considering random walks *with restart* on hypergraphs. Stephan and Zhu [154] and Chodrow et al. [40] considered *non-backtracking walks* to mitigate localization, where walkers tend to get trapped in short loops or high-degree neighborhoods.

M12. Personalized PageRank. PageRank (see P23) is a walk-based measure that evaluates node importance as the stationary distribution of a random walk with teleportation, where the walker randomly jumps to a preferred set of nodes with some probability. Takai et al. [156] extended *personalized PageRank* to hypergraphs by defining the random walk directly on the hypergraph structure, rather than relying on a pairwise projection. In this formulation, the teleportation distribution can be customized to prioritize certain nodes, enabling personalization of the ranking process. Piao et al. [129] considered PageRank for *directed hypergraphs*, where each hyperedge has explicitly defined head and tail node sets inside it.

M13. Vector centrality. Kovalenko et al. [91] proposed *vector centrality*, which assigns each node a *vector* (instead of a *scalar* in usual cases) of centrality scores. The dimension of this vector corresponds to the number of hyperedge sizes of interest, with each component of the vector quantifying the node's involvement in interactions of a specific size. Specifically, after computing the eigenvector centrality of all hyperedges (see M10), for each node, the element in the vector corresponding to each hyperedge size k is the sum of the eigenvector centrality values of all its incident hyperedges (see P2) of size k. The vector centrality thus captures a node's importance depending on the size of the interactions (i.e., hyperedges).

M14. Z/H-eigenvector centrality. Benson [17] defined three variants of eigenvector centrality measures: C/Z/H-eigenvector centrality. Among them, C-eigenvector centrality is equivalent to *eigenvector centrality* computed on the clique expansion (see M10). The other two variants, *Z-eigenvector centrality* (ZEC) and *H-eigenvector centrality* (HEC) are directly computed on r-uniform hypergraphs (see P8) using the *hypergraph adjacency tensor*. ZEC and HEC adapt the idea of eigenvector centrality to hypergraphs by working directly with their multi-way connections, using tensor-based spectral definitions that differ in how the eigenvalues are defined. See also, e.g., [87, 88], for general tensor-based methods for identifying important nodes in hypergraphs. A key limitation of both ZEC and HEC is that they are defined only for uniform hypergraphs.

M15. Uplifted eigenvector centrality. Contreras-Aso et al. [45] proposed *uplifted eigenvector centrality* as a tensor-based spectral approach for extending the aforementioned H-eigenvector centrality (see M14) to non-uniform hypergraphs. The method transforms a non-uniform hypergraph into a uniform one via an uplift process, in which smaller hyperedges are augmented with auxiliary nodes and larger hyperedges may be projected onto subsets of a chosen size.

M16. Nonlinear eigenvector centrality. Tudisco and Higham [161] introduced a spectral framework that jointly ranks nodes and hyperedges in a hypergraph using a mutually reinforcing rule: Nodes are important when they belong to important hyperedges, and hyperedges are important when they contain important nodes. The framework is flexible because it lets us choose transformation functions that specify how influence is pooled (e.g., sum-like, product-like, or other combinations). With suitable choices, it can reproduce classic eigenvector centrality (see M10), generalize Z-eigenvector centrality (see M14), or create new variants for non-uniform hypergraphs.

M17. Sub-hypergraph centrality.[3] Estrada and Rodríguez-Velázquez [52] extended the notion of subgraph centrality [51] from graphs to hypergraphs, defining *sub-hypergraph centrality* that measures the importance of each node by counting closed walks it is involved in, with shorter walks contributing more. A closed walk is a walk (see P11) starting and ending at the same node.

3.4 Subhypergraph-based Measures

Another important class of measures is based on *subhypergraphs* (see P6). Subhypergraph-based measures typically first identify central subhypergraphs, and then assume that nodes/hyperedges participating in a larger number, or more prominent, central subhypergraphs are more central and important, and thus assigned higher centrality scores.

M18. Hypercoreness. Leng et al. [101] generalized the notion of k-cores [147] to hypergraphs. The k-*hypercore* is the maximal subhypergraph (see P6) in which every node has degree (see P3) at least k. The *hypercoreness* of a node or hyperedge is then the largest k such that it belongs to the k-hypercore. Several variants refine this basic model: Luo et al. [115] required hyperedges of size above a threshold, Arafat et al. [8] defined cores via neighbor-degrees (see P4), and Kim et al. [80] added additional conditions on node co-occurrences. Some other researchers [26, 81, 106] extended the notion of subhypergraphs, proposing hypercore models in which hyperedges can be subsets of the original ones, rather than being preserved entirely as required in the typical definition of subhypergraphs (see P6).

M19. Hypertrussness. Wang et al. [170] and Qin et al. [136] generalized the notion of k-trusses [44] to hypergraphs. With a similar spirit as hypercoreness (see M18), a k-*hypertruss* is the maximal subhypergraph (see P6) in which each hyperedge participates in at least k *triangles*, and the *hypertrussness* of a node or hyperedge is the largest k for which it belongs to such a subhypergraph. Different definitions of triangles lead to different models: Wang et al. [170] defined a triangle as a triplet of nodes where each pair of nodes co-occurs in at least one hyperedge while Qin et al. [136] defined a triangle as a triplet of hyperedges that intersect pairwise but have an empty three-way intersection. Other notions of triangles in hypergraphs (see, e.g., [19, 90, 103]) can likewise be adopted to define hypertrussness.

M20. Hitting-set score Amburg et al. [6] proposed *hitting-set score* that measures the likelihood of each node being in the hitting set. The hitting set of a hypergraph is a subset of nodes that intersect every hyperedge. To estimate which nodes belong to the hitting set, the authors

[3] Despite its name, this measure only uses information about paths and is thus path-based instead of subhypergraph-based.

repeatedly applies a greedy set-cover heuristic, prunes the output to a minimal hitting set, and takes the union across many iterations. The score of a node is defined as the proportion of runs in which it appears in these minimal hitting sets. Nodes that are structurally indispensable for intersecting hyperedges thus accumulate higher scores.

M21. Core-periphery score. Both Tudisco and Higham [162] and Papachristou and Kleinberg [127] proposed methods for assigning *core-periphery score* in hypergraphs, aiming to quantify how "core" (i.e., central) each node is. These measures assume the existence of a subhypergraph consisting of *core* nodes that frequently participate across different groups, contrasted with more peripheral nodes that interact mainly through the core. Tudisco and Higham [162] formulated the problem as a nonlinear spectral optimization. Papachristou and Kleinberg [127] instead introduced a generative random hypergraph model in which the probability of each hyperedge depends on the nodes' core-periphery scores, and inference recovers the most likely scores explaining the observed hypergraph.

M22. Motif counts. The above subgrapph-based measures assume that nodes/hyperedges participating in *more prominent* central subhypergraphs are more important, while one can also consider the *frequency* of the participation in central subhypergraphs to quantify importance. Following such an idea, Lee et al. [99] proposed *hypergraph motifs* and used the *motif counts* of each node/hyperedge as a centrality measure. Since there are multiple motifs, each node/hyperedge is associated with a centrality vector instead of just a scalar. Other definitions of hypergraph motifs (see, e.g., [76, 82, 111]) can similarly be used to define motif-based centralities.

M23. Clustering coefficients. In pairwise graphs, the clustering coefficient (or transitivity) [171] measures the tendency of a node's neighbors to form triangles, capturing its role in cohesive groups. Several works have generalized this idea to hypergraphs by defining clustering coefficients for pairs of nodes [62, 85, 96, 120, 128] or for pairs of hyperedges [62, 83, 85, 86, 160, 188]. Aggregating these pairwise values yields centrality scores for individual nodes or hyperedges. These measures are typically normalized as fractions, although unnormalized (count-based) versions are also used when absolute participation is of interest.

3.5 Hybrid Measures

Below, we introduce *hybrid structural measures* based on multiple structural elements. Such measures typically capture the importance of nodes/hyperedges from various perspectives, by aggregating multiple measures. Under the high-level framework of hybrid measures, one can combine different measures under various aggregation functions.

M24. Gravity centrality. Inspired by Newton's law of universal gravitation, Xie et al. [178] introduced *gravity centrality* based on both *degrees* and *paths*. It measures the importance of each node v by considering its connections to all other nodes u, weighting each connection by the degrees of both nodes and inversely by the square of their distance $\delta(u,v)$, i.e., $\sum_{u \neq v \in \mathcal{V}} \frac{\deg(u)\deg(v)}{\delta(u,v)^2}$. The authors also considered a semi-local version, where for each node v, we set a distance threshold and only consider the other nodes u within this threshold. The authors specifically used distances in the s-line expansions (more precisely, a weighted sum of distances in s-line expansions with different s values; see P16), while in principle, as discussed in Section 2.2 and above, any definition of distances would suffice to give a reasonable definition. Wang et al. [168] extended this idea to further consider hyperedge-level and community-level interactions.

M25. Complex centrality. Zhou et al. [187] proposed *complex centrality* based on both *degrees* and *subhypergraphs*. It measures the importance of each node v with a mixture of degree centrality (see M1) and hypercoreness (see M18). Specifically, the authors use a Euclidean aggregation. The centrality core of each node v is $\sqrt{d_v^2 + k_v^2}$, where d_v is the degree centrality of v, and k_v is the hypercoreness of v. Guo et al. [66] have extended the idea to multilayer hypergraphs.

M26. Multi-centrality. Wu et al. [174] proposed *multi-centrality* for nodes based on *degrees*, *paths*, and *subhypergraphs*. Specifically, they combine degree centrality (see M1), betweenness centrality (see M6), and hypercoreness (see M18) into an integrated centrality measure, with an adaptive weighting scheme based on entropy.

M27. Multi-criteria centrality Xiao [176] proposed *multi-criteria centrality* that integrates *degree-based* and *path-based* information. For each node v, its importance is defined as a weighted sum of three components: (1) its overall degree (see P3 and M1), (2) its degree in a local subhypergraph around v, and (3) its betweenness (see P20 and M6).

M28. Improved PageRank Chen et al. [36] proposed *improved PageRank*, which combines *walk-based* PageRank with *degree-based* information. Globally, they introduce an improved PagaRank using node-similarity information. Locally, they quantify node importance via information entropy of neighbor degree distributions, with the high-level idea that a node is more influential if its connections are not only numerous but also diverse (see similar ideas in M9). Those two components are then combined to give the final measure.

4 Functional Measures

Functional measures assess importance going beyond static structural features, based on the role nodes or hyperedges play in the overall *functioning* of the hypergraph. They quantify centrality in terms of system behavior: how much the network's connectivity or robustness is disrupted when a node/hyperedge is perturbed, or how much value a node/hyperedge contributes across different coalitions of multiple nodes/hyperedges. We group these approaches into two sub-categories: *perturbation-based measures*, which evaluate structural changes caused by removing nodes or hyperedges, and *coalition-based measures*, which allocate importance using cooperative game-theoretical formulations.

4.1 Perturbation-based Measures

Perturbation-based measures typically assume that nodes/hyperedges whose removal leads to greater structural disruption are more central and important, and thus assigned higher centrality scores.

M29. Higher-order Von Neumann entropy. Hu et al. [69] proposed a centrality measure called *higher-order Von Neumann entropy*, which quantifies the importance of each hyperedge based on the change in the von Neumann entropy of the Laplacian matrix of its line expansion (see P16) after removal.[4] The more the von Neumann entropy decreases after a hyperedge is removed, the more central the hyperedge is considered. The intuition is that higher von Neumann entropy often corresponds to denser structures. The centrality value of each node is computed by aggregating the values of its incident hyperedges (see P2).

M30. Total loss. Xiao et al. [177] proposed a centrality measure called *total loss*, which quantified the importance of each node based on the decrease (i.e., "total loss") in the connectivity of the hypergraph. Specifically, the connectivity here is a function of the distances between all node pairs in the hypergraph, where shorter distances give higher connectivity.

M31. Grounded-Laplacian eigenvalue. Li et al. [105] proposed *grounded-Laplacian eigenvalue* to rank hyperedges by considering the effects of their removal, which combines *walk-based* spectral information with *perturbation-based* removal effects. For each hyperedge e, they remove e and all its constituent nodes (see P2), obtain the corresponding Laplacian of the remaining hypergraph

[4] The Laplacian matrix of a pairwise graph (here, the line expansion) is defined as $\mathbf{L} = \mathbf{D} - \mathbf{A}$, where $\mathbf{D}$ is a diagonal matrix of node degrees with $\mathbf{D}_{ii}$ being the degree (see P3) of node v_i, and A is the graph's adjacency matrix (see P21).

(the authors call it the *grounded Laplacian* of e), and then compute its smallest eigenvalue.[5] This measure captures the structural vulnerability induced by removing each hyperedge, with larger values indicating greater importance.

Alternative metrics to quantify the structural change

In the above measures (see M29 to M31), the Von Neumann entropy, the connectivity function, and the smallest eigenvalue of Laplacian, are the specific choices of the authors. In principle, any measure that quantifies the connectivity [92] or robustness [61] of hypergraph structure can be used instead.

4.2 Coalition-based Measures

Similarly to perturbations, *coalitions* can also be used to define centrality measures. Rather than focusing only on the presence or absence of individual nodes/hyperedges, coalition-based methods evaluate the contribution of a node/hyperedge across all possible subsets of nodes/hyperedges. This perspective is grounded in cooperative game theory, where the centrality of a node/hyperedge is often defined via its *Shapley value* [148], i.e., the expected marginal contribution it makes when joining a randomly ordered coalition.

M32. Clique-induced Shapley value. Huang and Tur [71] proposed *clique-induced Shapley value*, a coalition-based centrality based on a cooperative game where a coalition's value comes from the cliques it induces. Bigger cliques are given higher weights, and the corresponding Shapley value then fairly splits this clique-based value among nodes. This measure highlights nodes that tend to bridge overlapping communities.

M33. Line-expansion Shapley value. Roy and Ravindran [140] proposed *line-expansion Shapley value*, a coalition-based centrality for hypergraphs by first converting them into a weighted graph using line expansion (see P16). A cooperative game is then played on this weighted graph, using the Shapley value to determine each hyperedge's importance, which is finally shared equally among its constituent nodes.

5 Contextual Measures

Both structural measures (see Section 3) and functional measures (see Section 4) defined centrality and importance measures merely based on hypergraph topology. *Contextual measures* in contrast, extend beyond topology by incorporating additional information associated with nodes or hyperedges. Such information may include *features* (e.g., attributes or embeddings describing entities) or *labels* (e.g., roles or annotations specifying functional positions). These measures capture importance as a joint effect of structure and context, allowing centrality to reflect not only how elements are embedded in the hypergraph but also what characteristics they carry. We group existing approaches into three sub-categories: *feature-based*, *label-based*, and *hybrid* contextual measures.

5.1 Feature-based Measures

Feature-based measures incorporate non-structural attributes of nodes or hyperedges into the assessment of centrality and importance. These approaches combine structural information with external descriptors such as embeddings, metadata, or other feature vectors. The underlying

[5] Here, the Laplacian matrix is also defined as $\mathbf{L} = \mathbf{D} - \mathbf{A}$, but $\mathbf{D}$ and $\mathbf{A}$ are defined differently. Specifically, $\mathbf{D}$ is the diagonal matrix of neighbor-degrees (see P4), and $\mathbf{A}$ is a weighted adjacency matrix where each entry $\mathbf{A}_{ij}$ is the number of hyperedges containing both nodes v_i and v_j.

assumption is that centrality or importance in real-world hypergraphs depends not only on position within the topology but also on the characteristics entities carry. Feature-based measures therefore capture centrality as an interplay between structural prominence and attributive relevance.

M34. Structure-and-embedding score Chang et al. [33] proposed *structure-and-embedding score*, which combines *degree-based* structural scores and a *feature-based* score using learned node embeddings. The key idea is that central nodes are both structurally well-connected and distinctive in their learned representations. Structurally, they define two measures: *common-node centrality*, capturing how many nodes hyperedges share, and *common-hyperedge centrality*, capturing how many hyperedges nodes share. Embedding-wise, a *distance-based score* is computed in the learned embedding space, where nodes with large distances to others are considered more important.

M35. Representation-based influence score. Ni et al. [124] proposed *representation-based influence score*, which assigns scores to nodes by learning from diffusion outcomes. Rather than simulating influence spread and structural disruption for every node, the method uses results from a small set of representative nodes as supervision. A learning framework then generalizes these outcomes, producing scores that approximate each node's impact on information spreading and network connectivity. The resulting scores align with simulation-based influence while being far more efficient and transferable across different hypergraph structures.

5.2 Label-based Measures

Label-based measures exploit categorical annotations that specify the roles or functions of nodes within hyperedges. In contrast to feature-based approaches, which rely on continuous attributes or embeddings, label-based measures emphasize how role assignments shape importance. They assess centrality by considering not only where nodes or hyperedges are positioned in the structure, but also what roles they occupy in group interactions. This perspective is particularly relevant in settings such as collaboration, communication, or Q&A forums, where the influence of a participant depends on their position *and* their functional role.

M36. Edge-dependent role score. Choe et al. [42] proposed *edge-dependent role score* that combines structural cues from traditional measures with role-based attributes (e.g., first/last author, asker/answerer). The model learns attention weights and role probabilities within each hyperedge, then aggregates them to produce a score capturing both structural position and role-specific relevance. As a related effort, Bu et al. [27] introduced the concept of *group anchors*, where each hyperedge has a single but edge-dependent anchor (i.e., a node may be the anchor in one group but not in another), and modeled this through *anchor scores* on node-hyperedge pairs indicating the likelihood of anchorship.

M37. Matrix centrality. Vasilyeva et al. [163] proposed *matrix centrality* for annotated hypergraphs [39, 42] where nodes may occupy different roles within hyperedges. It extends vector centrality (see M13), which represents each node's importance as a vector indexed by hyperedge size, to a two-dimensional matrix representation in which centrality values are indexed jointly by hyperedge size and node role. Aggregating over the role dimension recovers the original vector centrality (see M13), while aggregating over hyperedge sizes produces *role centrality*, a complementary measure that captures a node's overall importance by functional position regardless of group size.

5.3 Hybrid Measures

Hybrid contextual measures combine both node or hyperedge *features* and *labels* when defining importance. By leveraging continuous attributes together with categorical role information, these approaches produce scores that reflect not only structural prominence but also how entities with particular characteristics and roles contribute to hypergraph interactions.

M38. Hypergraph attention score. In hypergraph neural networks, attention mechanisms (see, e.g., [12, 31, 104, 141]) are often used to weight nodes, hyperedges, or higher-order substructures during message passing. The resulting *hypergraph attention score* indicates which elements are more important for the specific learning task at hand, and can therefore be interpreted. These scores reflect importance jointly from connectivity, features, and labels, tailored to the supervised or self-supervised objective.

M39. Identity-aware score. Chen et al. [37] proposed *identity-aware score*, a hyperedge-level measure that learns group importance from real-world features (e.g., votes, citations, and views) while accounting for members' different roles. Role-specific attention and degree-based positional context inject attributes into message passing, and iterative node-hyperedge propagation produces a score capturing both structural position and role contributions.

6 Empirical Insights

In this section, we present the results of empirical analyses designed to provide insights into representative measures. Specifically, we summarize key findings from experiments that examine (1) the empirical (dis)similarity among different measures, and (2) the empirical computation time of measures. For reproducibility, we provide the source code and datasets utilized in our analyses in `https://github.com/jaewan01/hypergraph-centrality-survey`.

6.1 Experimental Settings

We describe the experimental settings, including machines, datasets, analyzed measures, and evaluation metrics.

Machines. All experiments are conducted on a workstation with AMD Ryzen 7 3700X and 128GB memory.

Datasets. We conduct our empirical analyses on ten real-world hypergraphs from five different domains:

- **Bills domain (senate-bills, house-bills [41, 57, 58]):** Each node represents a member of the U.S. Congress, and each hyperedge corresponds to a set of co-sponsors of a bill.
- **Email domain (email-enron, email-eu [19, 102, 180]):** Each node denotes an email address, and each hyperedge consists of the sender and all recipients of a single email.
- **Drug domain (ndc-classes, ndc-substances [19]):** Each hyperedge represents a drug. For *ndc-classes*, nodes in a hyperedge correspond to the class labels applied to the corresponding drug, and for *ndc-substances*, nodes in a hyperedge represent the substances comprising the corresponding drug.
- **Contact domain (contact-primary-school, contact-high-school [19, 119, 153]):** Each node represents an individual, and each hyperedge is a group of people who interacted within a given time interval.
- **Q&A domain (tags-ask-ubuntu, threads-ask-ubuntu [19]):** Each hyperedge represents a Q&A post. For *tags-ask-ubuntu*, nodes in a hyperedge correspond to the tags associated with the corresponding post, and for *threads-ask-ubuntu*, nodes in a hyperedge correspond to the users participating in the corresponding post.

For all the hypergraphs, we only consider hyperedges of size greater than or equal to two, and extract the largest connected component of each hypergraph. We report the basic statistics of the preprocessed hypergraphs in Table 1.

Table 1: Basic statistics of the real-world hypergraphs after preprocessing used in our empirical analyses. The numbers of nodes and hyperedges are denoted by $n = |\mathcal{V}|$ and $m = |\mathcal{E}|$, respectively. The average and maximum sizes of hyperedges are denoted by $\text{avg}_{e\in\mathcal{E}}|e|$ and $\max_{e\in\mathcal{E}}|e|$, respectively. The density [70] is defined as m/n, and the overlapness [98] is defined as $\sum_{e\in\mathcal{E}}|e|/n$.

Dataset (Abbreviation)	$n = \|\mathcal{V}\|$	$m = \|\mathcal{E}\|$	$\text{avg}_{e\in\mathcal{E}}\|e\|$	$\max_{e\in\mathcal{E}}\|e\|$	Density	Overlapness
senate-bills (SB)	294	29,157	7.96	99	99.17	789.62
house-bills (HB)	1,494	60,987	20.47	399	40.82	835.79
email-enron (EEN)	143	10,454	2.53	37	73.10	185.20
email-eu (EEU)	986	209,508	2.56	40	212.48	544.47
ndc-classes (NDCC)	628	39,717	3.46	39	63.24	218.74
ndc-substances (NDCS)	3,414	28,506	4.95	187	8.35	41.36
contact-primary-school (CTP)	242	106,879	2.10	5	441.65	925.61
contact-high-school (CTH)	327	172,035	2.05	5	526.10	1078.65
tags-ask-ubuntu (TAU)	3,021	219,076	3.11	5	72.52	225.80
threads-ask-ubuntu (THU)	82,048	113,575	2.31	14	1.38	3.20

Analyzed Measures. We employ ten representative hypergraph centrality and importance measures. We exclude measures that (1) depend on user-specified hyperparameters (e.g., M3, M8, and M32), (2) produce vector-valued outputs rather than a single scalar score (e.g., M13 and M22), and (3) requires external labels or features (i.e., contextual measures described in Section 5). The evaluated measures are categorized according to our proposed taxonomy as follows:

- **Degree-based measures:** Degree centrality (M1), neighbor-degree centrality (M2), and line-expansion degree centrality (M4).
- **Path-based measures:** Closeness centrality (M5), betweenness centrality (M6), and harmonic centrality (M7).
- **Walk-based measures:** Eigenvector centrality (M10), random-walk centrality (M11), and uplifted eigenvector centrality (M15).
- **Subhypergraph-based measure:** Hypercoreness (M18).

We evaluate measures defined for nodes and those defined for hyperedges separately. Among the analyzed measures, neighbor-degree centrality and uplifted eigenvector centrality are defined only for nodes, whereas line-expansion degree centrality is defined only for hyperedges. All the other measures are defined for both nodes and hyperedges. All implementations are based on the XGI library [93]. The source code and datasets used in this section are available at `https://github.com/jaewan01/hypergraph-centrality-survey`.

Evaluation Metrics. To quantitatively assess the (dis)similarity between different measures, we use four evaluation metrics from two complementary perspectives: two *global metrics* and two *top-k metrics*.

- **Global metrics:** We first assess the global (dis)similarity between each pair of measures using the **Pearson correlation coefficient** and the **Spearman rank correlation coefficient**.[6] Both correlation coefficients range from -1 to 1, where -1 indicates a perfect inverse relationship between the two measures, while 1 indicates a perfect agreement.

[6] Given two centrality score vectors $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$, the Pearson correlation coefficient is defined as: $\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}}$, where $\bar{x}$ and $\bar{y}$ denote the mean values of $\mathbf{x}$ and $\mathbf{y}$, respectively. The Spearman rank correlation coefficient is computed as the Pearson correlation between the rank-transformed vectors of $\mathbf{x}$ and $\mathbf{y}$.

- **Top-k metrics:** To analyze the consistency among highly ranked nodes, we further employ the **Jaccard similarity** and the **top-k overlap ratio**.[7] We calculate both metrics using the top 5% of the ranked nodes or hyperedges. To handle potential ties in rankings, we follow the procedure in Webber et al. [172], including all tied items at rank k in each set. Both metrics take values in the range $[0, 1]$, where 0 indicates no agreement between the two measures, while 1 indicates complete alignment.

6.2 Correlation Analysis

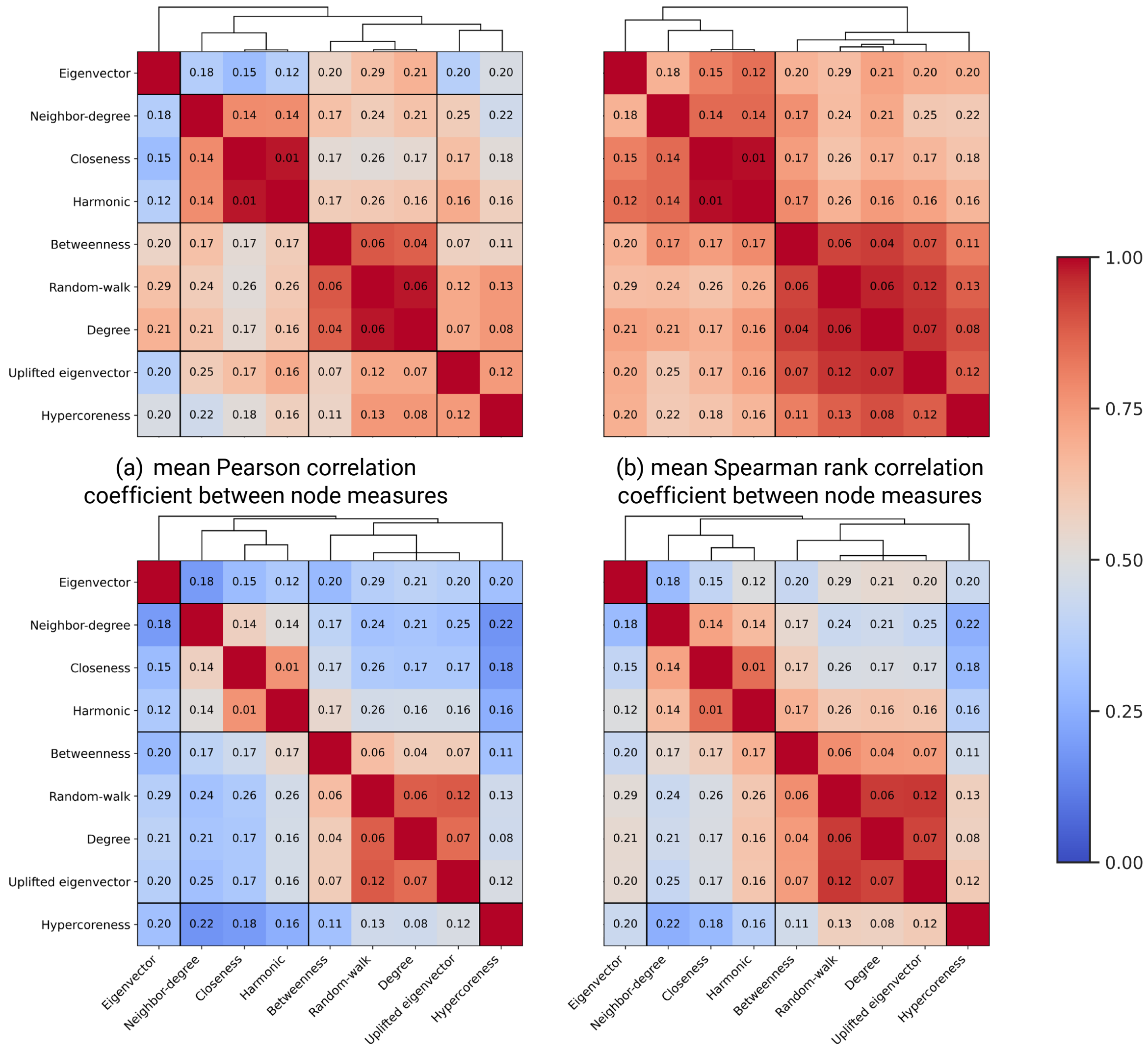
In this section, we present an empirical correlation analysis on different measures. Specifically, we evaluate the measures described in Section 6.1 across ten real-world hypergraphs, using the four evaluation metrics introduced in the same section. In Figs. 3 and 4, we present the mean value of each correlation metric for node and hyperedge measures, respectively, with standard deviations across datasets annotated in each cell. Additionally, we apply hierarchical clustering to each metric, where correlation scores are linearly transformed into distances (0 indicating perfect similarity and 1 indicating complete dissimilarity). The resulting dendrograms are displayed above each colormap, and we further report the discrete cluster assignments that yield the highest silhouette score. For both node and hyperedge measures, we derived three consistent key take-home messages regarding the results:

High global correlation, limited top-ranked agreement. Across all datasets, both node and hyperedge measures exhibit strong global correlations, with average Pearson and Spearman rank correlation coefficients of 0.627 and 0.773 for nodes, and 0.400 and 0.434 for hyperedges, respectively. Meanwhile, the agreement among highly ranked elements provides a complementary perspective: the average Jaccard similarity and top-k overlap ratio for the top 5% of elements are 0.443 and 0.568 for nodes, and 0.271 and 0.355 for hyperedges. Note that the global correlation metrics range from $[-1, 1]$, whereas the top-k metrics range from $[0, 1]$ and quantify a different aspect of agreement. Together, these results suggest that while different measures capture broadly consistent overall importance trends, they often diverge in identifying the most influential nodes or hyperedges within each hypergraph.

Consistent clusters across metrics. Hierarchical clustering based on the similarity matrices consistently partitions both node and hyperedge measures into a small number of stable groups across all metrics. For node centralities, two clear clusters consistently appear based on the silhouette score: (1) neighbor-degree, closeness, and harmonic centralities, and (2) betweenness and degree centralities. Within these clusters, the average similarities are notably higher than the overall averages. Specifically, the average Pearson and Spearman correlation coefficients reach 0.848 and 0.898 for Cluster 1, and 0.874 and 0.934 for Cluster 2, while the overall averages are 0.627 and 0.773, respectively, Similarly, for the top 5% of nodes, Jaccard similarities are 0.616 and 0.594, and top-k overlap ratios are 0.742 and 0.745 within Cluster 1 and Cluster 2, respectively, while the overall averages are 0.443 and 0.568.

A similar pattern is observed for hyperedge centralities, where the overall average is characterized by Pearson and Spearman rank correlation coefficients of 0.400 and 0.434, and by Jaccard similarity and top-k overlap ratio (for the top 5% of hyperedges) of 0.271 and 0.355, respectively. Here, the two clusters are (1) closeness and harmonic centralities, and (2) degree and random-walk centralities. In terms of average Pearson and Spearman correlation coefficients, Cluster 1 achieves 0.980 and 0.976, respectively, and Cluster 2 achieves 0.954 and 0.729, respectively. For Jaccard similarity and top-k overlap ratio, Cluster 1 achieves 0.856, 0.909, and Cluster 2 achieves 0.649, 0.741, respectively.

[7] Let $S_x^{(k)}$ and $S_y^{(k)}$ denote the sets of top-k nodes identified by two different measures. The Jaccard similarity and the top-k overlap ratio are defined as $\frac{|S_x^{(k)} \cap S_y^{(k)}|}{|S_x^{(k)} \cup S_y^{(k)}|}$ and $\frac{2 * |S_x^{(k)} \cap S_y^{(k)}|}{|S_x^{(k)}| + |S_y^{(k)}|}$, respectively.



(a) mean Pearson correlation coefficient between node measures

(b) mean Spearman rank correlation coefficient between node measures

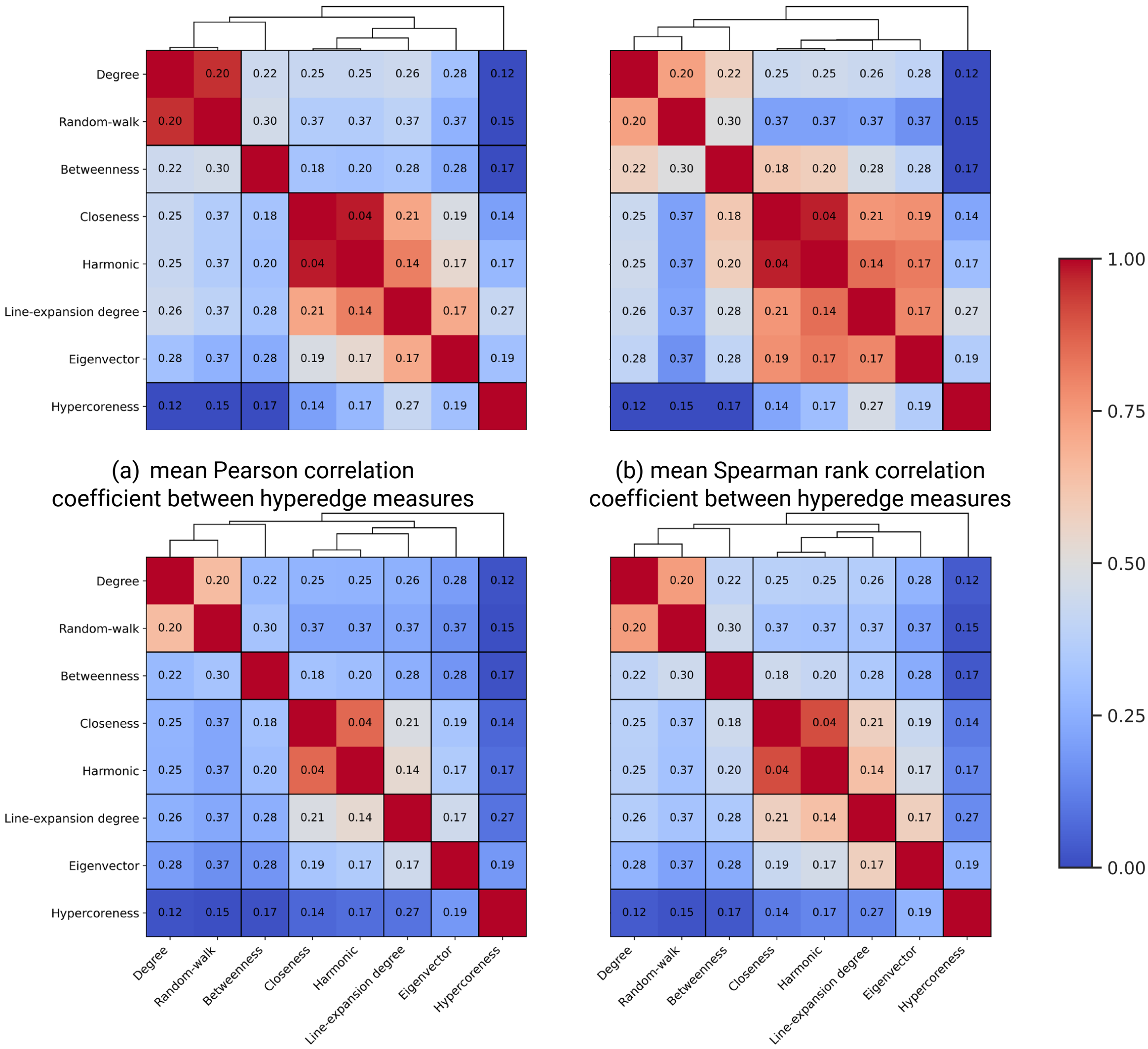
(c) mean Jaccard similarity @ 5% between node measures

(d) mean top-k overlap ratio @ 5% between node measures

Figure 3: Correlation between node centrality and importance measures in terms of (a) Pearson correlation coefficient, (b) Spearman rank correlation coefficient, (c) Jaccard similarity @ 5%, and (d) top-k overlap ratio @ 5%. The colormap indicates the average value of each measure across all datasets, and we annotate the standard deviation across datasets in each cell.

These results confirm that certain groups of measures exhibit strong internal agreement across multiple similarity metrics, reflecting shared measurement foundations (e.g., harmonic centrality is a variant of closeness centrality designed for disconnected hypergraphs). In contrast, other measures maintain distinct computational or conceptual perspectives.

Distinct singleton measures. While most measures form consistent clusters across metrics, certain measures behave as distinct singletons, exhibiting low similarity with all others. For nodes, eigenvector centrality consistently forms a singleton cluster, with average correlations to the other measures of 0.470 (Pearson correlation coefficient), 0.722 (Spearman rank correlation coefficient), 0.321 (Jaccard similarity), and 0.454 (top-k overlap ratio). For hyperedges, hypercoreness acts as a singleton across all metrics, with corresponding averages of 0.136, 0.116, 0.066, and 0.107. These values are substantially lower than the overall averages (Pearson correlation coefficient 0.627 and 0.400, Spearman rank correlation coefficient 0.773 and 0.434, Jaccard similarity 0.443 and 0.271, and top-k overlap ratio 0.568 and 0.355 for nodes and hyperedges,



(a) mean Pearson correlation coefficient between hyperedge measures

(b) mean Spearman rank correlation coefficient between hyperedge measures

(c) mean Jaccard similarity @ 5% between hyperedge measures

(d) mean top-k overlap ratio @ 5% between hyperedge measures

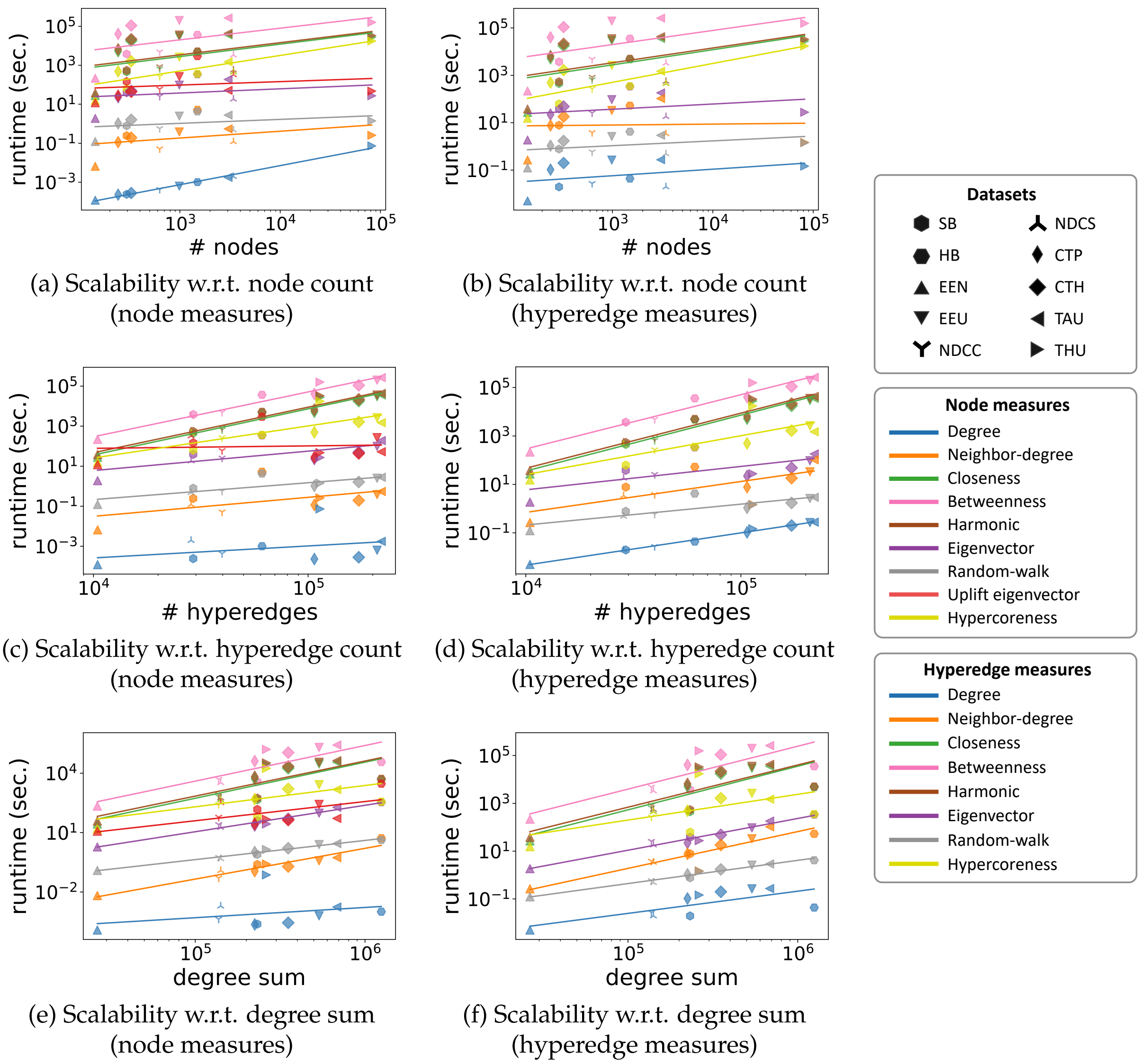
Figure 4: Correlation between hyperedge centrality and importance measures in terms of (a) Pearson correlation coefficient, (b) Spearman rank correlation coefficient, (c) Jaccard similarity @ 5%, and (d) top-k overlap ratio @ 5%. The colormap indicates the average value of each measure across all datasets, and we annotate the standard deviation across datasets in each cell.

respectively).

This divergence indicates that these singleton measures capture structural characteristics that are largely independent of those reflected by other measures. For example, hypercoreness identifies dense subhypergraph participation, offering unique perspectives on higher-order connectivity that are not well represented by conventional degree-, path-, or walk-based measures.

6.3 Computation Time Analysis

In this section, we analyze the empirical computation time of different measures described in Section 6.1. We measure the total computation time required to calculate the centrality and importance scores for all nodes or hyperedges, across the ten real-world hypergraphs. All experiments are performed under the same computational environment, and the results are compared across measures and datasets to assess scalability and practical feasibility. We present the results in Fig. 5, evaluating the computation time with respect to three structural quantities:



(a) Scalability w.r.t. node count (node measures)

(b) Scalability w.r.t. node count (hyperedge measures)

(c) Scalability w.r.t. hyperedge count (node measures)

(d) Scalability w.r.t. hyperedge count (hyperedge measures)

(e) Scalability w.r.t. degree sum (node measures)

(f) Scalability w.r.t. degree sum (hyperedge measures)

Figure 5: Computation time of node and hyperedge measures with respect to different structural quantities in log-log scale. Each subfigure reports the total computation time with respect to (a) & (b) the number of nodes, (c) & (d) the number of hyperedges, and (e) & (f) the total degree sum, for node and hyperedge measures, respectively.

(1) number of nodes, (2) number of hyperedges, and (3) sum of degrees (i.e., $\sum_{v \in \mathcal{V}} \deg(v)$). From the results, we derive three key take-home messages:

Empirical computation time decreases from path-based to walk- and degree-based measures. The computation time of a measure strongly depends on the structural element it utilizes, as categorized in our proposed taxonomy. In general, the computational cost in terms of time follows the order:

$$\text{path-based} > \text{subhypergraph-based} > \{\text{walk-based \& degree-based}\}$$

Moreover, based on the slopes of the regression lines in Fig. 5, we further examine the empirical scalability of each measure. For every measure, we compute the slope of the regression line between computation time and three structural quantities—the number of nodes, the number of hyperedges, and the degree sum—and take the largest slope among them as its representative scaling factor. We classify scalability into three categories according to this slope: *sublinear*

(< 1), *subquadratic* ($1 \leq$ slope < 2), and *superquadratic* (≥ 2). The resulting empirical scalability, grouped by the utilized structural element, is summarized as follows:

- **Path-based (superquadratic).** Path-based measures—closeness, betweenness, and harmonic centralities—exhibit the highest computational complexity due to their reliance on all-pairs or multi-step distance computations. These measures show superquadratic scaling, indicating rapidly increasing computation time with hypergraph size.
- **Subhypergraph-based (subquadratic).** The subhypergraph-based measure, hypercoreness, demonstrates moderately high computational cost, as it iteratively evaluates dense substructure membership. Its scaling slope remains below 2, suggesting subquadratic growth.
- **Degree- and walk-based (sublinear to subquadratic).** Among all measures, degree centrality is the most scalable, exhibiting sublinear growth on the log–log regression line. Walk-based measures—eigenvector, uplifted eigenvector, and random-walk centralities—are more efficient than path- and subhypergraph-based measures but still require iterative convergence. Empirically, they display subquadratic scaling, with slopes below approximately 1.4. Other degree-based measures, including neighbor-degree and line-expansion degree centralities, also fall within the subquadratic range, with slopes around 1.6.

Subhypergraph- and path-based measures can become computationally infeasible at scale. While the previous analysis characterizes the empirical scalability of measures in terms of their asymptotic growth with hypergraph size, here we examine the practical implications in terms of absolute computation time. We focus on the largest dataset, *tags-ask-ubuntu (TAU)*, which contains the greatest number of hyperedges among all datasets. In practice, degree- and walk-based measures, which exhibit sublinear to subquadratic scaling, complete computation within a few seconds to a few minutes even on TAU. The subhypergraph-based measure, hypercoreness, requires moderately more computation, completing in approximately four hours. In contrast, path-based measures—closeness, betweenness, and harmonic centralities—display superquadratic scaling and require from ten hours to several days of computation on the same dataset. These observations suggest that for million- or billion-scale hypergraphs, subhypergraph- and path-based measures may become computationally infeasible.

Degree-based measures can serve as cheaper proxies for expensive measures. By jointly considering the similarity analysis in Section 6.2 and the computation time results in this section, we identify practical substitutes for computationally expensive measures. For node measures, closeness and harmonic centralities show strong correlation with neighbor-degree centrality, while the latter is at least 905 times faster across datasets (the minimum speedup is observed on the *house-bills* dataset). Similarly, betweenness centrality correlates well with degree centrality, with degree centrality being at least 1.7 million times faster across datasets (the minimum speedup is observed on *ndc-substances*). Moreover, random-walk centrality correlates strongly with degree centrality while being more expensive, requiring at least 8.6 times longer computation across datasets (minimum observed on *contact-high-school*). Overall, these findings demonstrate that degree-based measures—in particular, degree and neighbor-degree centralities—can serve as scalable, low-cost proxies for more complex path- and walk-based measures, offering a practical balance between interpretability and computational efficiency.

7 Real-World Applications

Centrality and importance measures for hypergraphs provide powerful tools for analyzing systems in which interactions occur among groups rather than just pairs. Across diverse domains, such measures have been employed to identify influential individuals in social networks, key molecules in biological systems, and critical nodes in transportation and infrastructure networks. By quantifying how both nodes and hyperedges contribute to the organization, efficiency, and resilience of higher-order systems, hypergraph-based centrality and importance measures yield actionable insights that extend beyond the capabilities of traditional pairwise analysis. In this section, we review representative applications across social, biological, and transportation domains, illustrating how hypergraph formulations uncover central structures in complex real-world systems.

7.1 Social Networks

In social systems where individuals often engage in group interactions—such as event co-attendance, group chats, or multi-author collaborations—modeling the structure as a hypergraph enables a richer view of influence dynamics [175]. By applying centrality and importance measures designed for hypergraphs, one can better identify those individuals who are influential in group-based processes.

For instance, Mancastroppa et al. [118] and Bu et al. [26] showed that, in real-world social networks modeled as hypergraphs, nodes with high hypercoreness (see M18) serve as highly effective seeds for spreading and consensus processes. More recently, Zhang et al. [183] showed that distance-based fuzzy centrality (see M9) accurately ranks influential nodes under spreading dynamics. Hu et al. [69] showed that higher-order Von Neumann entropy (see M29) effectively identifies nodes that yield maximal influence and maintain network connectivity. Amato et al. [4] adapted degree (see M1), closeness (see M5), betweenness (see M6), and neighborhood centralities (see M8) to successfully identify “lurkers,” silent users who consume content without contributing, in online social networks.

Discussion. Contextual measures that consider external labels or features remain underexplored, while in many real-world social networks, e.g., social media platforms, side information from text, images, video, and profiles can complement topology by signaling credibility, affect, or topic alignment [5, 130].

7.2 Biological Systems

Many biological interactions are inherently polyadic: protein complexes involve several subunits, metabolic reactions require multiple substrates and enzymes, and gene regulation can depend on combinations of transcription factors and co-factors. Representing these as hypergraphs preserves the integrity of such multi-component interactions [122]. Hypergraph-based centrality and importance measures then help us spot which molecules play a key role in these multi-way systems, e.g., which protein’s removal would cause major disruption, or which gene strongly influences a cellular response.

For example, Feng et al. [54] used hypergraphs to model gene perturbations under viral infection and showed that the value of betweenness (see M6) well identifies critical response genes. Likewise, Barton et al. [13] built hypergraphs over gene-expression interactions and analyzed their structure via line-graph-based measures, including degree (see M1), closeness (see M5), betweenness (see M6), and eigenvector centralities (see M10), thereby identifying key genes and gene groups associated with distinct expression patterns. Moreover, Lawson et al. [95] applied nonlinear eigenvector centrality (see M16) to a yeast protein-complex hypergraph and showed that nodes with high centrality scores correspond to essential proteins and complexes. In a related biomedical application, Rafferty et al. [137] modeled multi-morbidity—the co-occurrence of multiple chronic health conditions—using a hypergraph constructed from patient health records, and applied eigenvector centrality (see M10) to identify the influential sets of co-occurring diseases.

Discussion. In many biological systems, the interactions have special structures, e.g., they may be directed or require exact ratios of participants [32]. Ideal measures for biological systems should take these facts into consideration.

7.3 Neuroscience Networks

The human brain is a complex system where cognitive functions—such as multisensory integration, decision-making, and high-level abstraction—emerge from the coordinated activity of multiple neural populations rather than simple pairwise communication. While traditional graph theory models the brain via dyadic links, this approach often fails to capture the synergy and redundancy inherent in neural computation [117]. Hypergraphs provide a rigorous framework for modeling these polyadic interactions, and the corresponding hypergraph centrality and importance measures offer novel insights into brain organization and pathology.

For example, Gu et al. [65] pioneered a functional hypergraph approach by treating edges as nodes and their covariance as hyperedges. Leveraging hypergraph incidence properties, e.g., degree centrality (see M1), they decomposed the connectome into clusters (functional cores), stars (localized drivers), and bridges (connectors), demonstrating that brain variance is largely driven by focal control centers rather than global waves. Furthermore, Santos et al. [142] applied eigenvector centrality (see M10) to 3-uniform hypergraphs constructed via interaction information. Their spectral analysis successfully distinguished between synergistic hubs (e.g., the Angular Gyrus) essential for integration and redundant hubs (e.g., the Motor Cortex) that ensure robust execution.

Discussion. While most current models are static, neural processing is fluid. Future work ideally requires the development of dynamic weighted hypergraph approaches that can track centrality fluctuations in real-time [185].

7.4 Transportation Networks

Transportation systems—including maritime shipping, rail networks, and multimodal transit—often involve group-wise interactions (e.g., multiple vessels calling at a hub port, multi-stop high-speed-rail services, or multimodal transfer hubs) that are naturally represented by hypergraphs [68, 131]. Hypergraph-based centrality and importance measures then help identify critical nodes or hubs whose disruption would significantly impair system connectivity and flow efficiency.

For example, Tocchi et al. [159] built a hypergraph of worldwide maritime container-transportation services and showed that betweenness (see M6) well indicates strategic global hubs whose removal would cause major fragmentation of the network. Likewise, Yin et al. [182] constructed a hypergraph of the Chinese high-speed rail system and evaluated station importance through multiple measures, including degree (see M1), betweenness (see M6), and closeness (see M5), demonstrating that those hypergraph-based centralities effectively capture both the structural and operational significance of key transfer nodes. Moreover, Yin et al. [181] modeled multimodal highway-railway-aviation transportation as a hypergraph and revealed that hubs with high betweenness (see M6) play a crucial role in maintaining connectivity under perturbations, providing a quantitative foundation for resilience assessment and infrastructure planning.

Discussion. Many transportation networks are driven by space and time [169]. Ideal measures for transportation networks should be able to incorporate both spatial and temporal information.

7.5 Broader Discussions

Across these domains, the applications reveal that *centrality* or *importance* is not a single, monolithic concept but is highly problem-dependent. A clear trend has emerged toward shifting from purely structural measures to learned, task-specific ones. In many state-of-the-art applications—particularly in data-rich areas such as traffic forecasting [116], recommender systems [110], and logistics (e.g., resource allocation and task scheduling) [151]—learning-based methods, especially hypergraph neural networks, have become increasingly prevalent. We expect this evolution to continue, with more contextual measures being used in real-world scenarios.

8 Conclusions and Future Directions

In this survey, we systematized 39 hypergraph centrality and importance measures (see Fig. 2) into a unified, three-way taxonomy: *structural* (Section 3), *functional* (Section 4), and *contextual* (Section 5) measures. Beyond mapping the space, we provided an empirical comparison of their similarities and computation time characteristics (Section 6), and reviewed representative applications across social, biological, and infrastructure domains (Section 7). This integrated view is intended to help researchers and practitioners select measures that align with their modeling assumptions, and to reveal principled avenues for new designs. Below, we outline several concrete opportunities that follow naturally from this survey.

Axiomatic foundations. There have been efforts on developing axioms for centrality and importance measures on pairwise graphs [24, 25]. Researchers may also develop axioms for measures on hypergraphs. Axiomatic foundations would be helpful for clarifying the fundamental principles that a measure should satisfy, enabling formal comparisons among different measures, and guiding the design of new ones that adhere to desirable properties, bridging theoretical rigor and practical interpretability.

Unified framework for paths and distances. As discussed in Sections 2.2 and 3.2, the notions of paths and distances have been generalized in various ways on hypergraphs [132, 164], which gives rise to different path-based measures. Researchers may try to provide a meta-framework that parameterizes and unifies different choices of definitions of paths and distances. This would allow practitioners systematically select or tune the most suitable notion for their task, compare measures under a common lens, and analyze how different modeling assumptions (e.g., transition rules, walk constraints, or hyperedge weights) influence centrality outcomes, facilitating fair evaluations and theoretical connections among diverse path-based measures.

Truly non-uniform spectral measures. Although Contreras-Aso et al. [45] have proposed uplifted eigenvector centrality (see M15) that enables spectral analysis on non-uniform hypergraphs by introducing auxiliary nodes to achieve uniformity, this approach effectively reduces the problem to a uniform case rather than handling non-uniformity natively. Researchers may aim to develop truly non-uniform spectral operators that directly incorporate variable hyperedge sizes into the spectral definition itself, ensuring well-posedness, convergence guarantees, and scalable solvers for joint node-hyperedge importance estimation.

Measures for generalized hypergraphs. Real-world systems are often better modeled as generalized hypergraphs, e.g., temporal [55, 97], directed [9, 121], and multilayer [135, 168] ones. Researchers may extend and propose measures for such generalized settings. In turn, such measures would allow practitioners to better capture the multi-aspect dynamics of real-world systems—from time-sensitive coordination in temporal networks to interdependent processes in multilayer infrastructures—broadening both the explanatory power and practical applicability of hypergraph-based analysis.

Fast approximation for expensive measures. Based on our empirical analyses in Section 6, some measures (e.g., path-based measures) are empirically heavy and computationally expensive. While several studies have proposed efficient approximation techniques for analogous measures on pairwise graphs [3, 138, 139], similar efforts for hypergraphs remain limited. Future research may design approximation schemes that preserve accuracy guarantees while drastically reducing computation time and memory costs. Leveraging hardware acceleration such as GPUs and distributed computing frameworks [48, 78] can further scale these methods to massive real-world datasets. Such advances would make heavy measures more accessible in practice, enabling their integration into real-time analytics, dynamic monitoring, and large-scale machine learning pipelines.

Standardized and comprehensive benchmarking. Our empirical analyses in Section 6 provide initial insights into the behaviors and relationships among various measures, but they remain limited in scope and preliminary in scale. Similar benchmarking efforts have been actively conducted for pairwise graphs [72, 143]. Future research may pursue more standardized and comprehensive benchmarking efforts that systematically evaluate a wide range of measures on hypergraphs across diverse datasets, domains, and hypergraph structures. Such benchmarking frameworks—ideally supported by open repositories, unified implementations, and reproducible protocols—would enable fairer comparisons, reveal domain-dependent strengths and weaknesses, and ultimately guide the development of more robust and practically useful centrality and importance measures.

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